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List of Publications and communicated papers

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3. Distribution free newsvendor model with consignment policy and retailer's royalty reduction. Submitted to *International Journal of Production Research.*

4. A multi-retailer supply chain model with backorder and variable production cost. Submitted to *RAIRO-Operations Research.*

5. Relation between quality of products and production rate in a single-vendor multi-retailer joint economic lot size model with variable production cost. Under review in *International Journal of Production Research.*

Contents

1 Introduction

The thesis entitled as 'Some problems on supply chain management' consists of recent development on supply chain management. Supply chain management is handling of the flow of goods, data, information, and fund among different stages of an extended industrial sector which includes manufacturers, vendors, retailers, customers or any other facilities. The target of a successful supply chain is to satisfy the end customer's demand. This thesis covers a number of problems which are the hurdles to form a successful supply chain. The thesis contains ten chapters. Chapter 1 and 2 consist of abstract and introduction, respectively. The mathematical models based on this research starts from Chapter 3. The first research model deals with a single-vendor single-buyer supply chain model with single-setup-multi-delivery (SSMD) policy. An effort to reduce the vendor's setup cost is developed to gain more at the optimum level. The second research model is an extended version of the first one with imperfect quality of products. The third model extends all previous models with setup cost reduction and quality improvement of products under just-in-time manufacturing system. The fourth model relaxed one assumption of previous models as constant demand throughout the year. This fourth model was based on price-dependent demand with fixed and variable purchase cost. The fifth research model contains of a different delivery policy named as consignment policy. The model is solved using a distribution free approach with known mean and standard deviation. The sixth model is constructed with a single-vendor multi-buyer supply chain model under variable production cost which is dependent on the production rate. The seventh research model extends the sixth one with the reliability of the production process. The eighth research model consists of a three-echelon facility location model. This study emphasizes on a comparative study among three different dimensional facility location problems.

1.1 Literature review

Goyal [1] developed the first research work on the integrated vendor-buyer problem. Banerjee [2] extended Goyal's [1] model with an assumption on the number of lot size. Goyal [3] extended Banerjee's [2] model by assuming the manufacturing quantity of the vendor as an integer multiple of the buyer's ordering quantity. Ha and Kim [4] developed an integrated single-retailer single-supplier model with lot splitting policy. An integrated lot splitting model deals with multiple shipments in small lots. Ouyang et al. [5] investigated an integrated vendor-buyer cooperative model with controllable lead time and stochastic demand. Shah et al. [6] developed an optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Sarkar et al. [7] derived an integrated vendor-buyer inventory model with controllable lead time and delay-in-payments.

Liao and Shyu [8] first incorporated a probabilistic inventory model assuming lead time as a unique decision variable. Ben-Daya and Rauf [9] considered an inventory model as an extension of Liao and Shyu's [8] model where lead time is one of the decision variables. Ben-Daya and Rauf's [9] model dealt with no shortage and continuous lead time. Ouyang et al. [10] extended Ben-Daya and Rauf's [9] model by assuming discrete lead time and shortages. Pan and Yang [11] analyzed an integrated inventory model with lead time in a controllable manner.

Scarf [12] first introduced the min-max distribution free approach in a newsvendor problem. Gallego and Moon [13] simplified Scarf's [12] ordering rule and made this approach very popular to the researchers. Ouyang et al. [14] investigated an inventory model with distribution free approach. They used an investment function to improve quality of products and to reduce the setup cost. Sarkar and Majumder [15] developed an integrated vendor-buyer supply chain model with distribution free approach and manufacturer's setup cost reduction.

Whitin [16] developed concepts of economic price theory and inventory control. Lau and Lau [17] extended the classical newsboy problem with stochastic price-demand relationship. Gallego and Ryzin [18] investigated dynamic pricing of inventories where demand is price-sensitive as well as stochastic and firm's objective is to maximize the expected profit. Abad [19] formulated a dynamic and lot size model for perishable items. Sana [20] developed an economic order quantity model for perishable items with quadratic price-sensitive demand.

In the classical supply chain model, the rate of production is assumed to be inflexible or constant. However, in many cases the machine production rate may easily change (Khouja and Mehrez [21]). Machine tool cost also increases with increasing production rate. According to the analysis of Conard and McClamrock [22], a 10% change in the processing rate results in a 50% change in the machine tool cost. Porteus [23] explained the gradual fall of the product quality with an increased amount of production. The process approaches to the *out-of-control* state from the 'in-control' state as the number of produced units increases. Rosenblatt and Lee [24] considered the elapsed time until the production process reaches the *out-of-control* state to be an exponentially distributed random variable.

The first research work was done by Weber [25] in his industrial location theory. It was extended by Hakimi [26]. The concept of supply chain management (SCM) was established by Weber and Oliver [27]. Chopra *et al.* [28] showed an excel based solution of facility location model. According to ReVelle *et al.* [29], future studies led to different location models such as analytic model, continuous model, discrete location model and network model. Sana [30] introduced an inventory model in supply chain environment. Teng *et al.* [31] developed a supply chain model where the optimal economic order quantity for buyer-distributor-vendor was derived without derivative.

Melo *et al.* [32] mentioned, in his review article that, six different groups of discrete facility location problem entitled as median problems, center problems, covering problems, uncapacitated facility location problems (UFLP), capacitated facility location problems (CFLP) and supply chain network design (SCND) problems. The first three problems were well discussed in Owen and Daskin's [33] paper. A two echelon supply chain network was introduced by Amiri [34]. Hinojosa *et al.* [35] studied a dynamic supply chain with inventory.

The research models developed in the thesis based on the various problems on supply chain management are described onwards.

2 Integrated vendor-buyer supply chain model with vendor's setup cost reduction

This study deals with an integrated vendor-buyer supply chain model. Two models are constructed based on the probability distribution of the lead time demand. The lead time demand follows a normal distribution in the first model. In the second model, we consider the distribution free approach for the lead time demand. For the second model, only mean and standard deviation are known. The aim of our model is to reduce the total system cost by considering the setup cost reduction of the vendor.

Assumptions

1. When the buyer orders a lot size Q, the vendor manufactures the lot mQ with finite production rate $P(P > D)$ at one setup but delivers the quantity Q over m times (Single setup multi-delivery policy).

2. The buyer places an order when the level of inventory reaches to the reorder point R.

3. The reorder point is $R = DL + k\sigma\sqrt{L}$, where DL = the expected demand during the lead time, $k\sigma\sqrt{L}$ = safety stock, and k = safety factor.

4. Shortages are allowed and fully backordered.

5. The lead time L has n mutually independent components. For the ith component, a_i = minimum duration, b_i = normal duration, and c_i = crashing cost per unit time. For the sake of convenience, we assume $c_1 \leq c_2 \leq ... \leq c_n$.

6. We assume $L_0 \equiv \sum_{j=1}^{n} b_j$ and L_i be the length of the lead time with components $1, 2, ...i$ crashed to their minimum duration, then L_i can be expressed as $L_i =$

$L_0 - \sum_{j=1}^{i}(b_j - a_j)$, $i = 1, 2, ...n$. The lead time crashing cost per cycle $C(L)$ is expressed as $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

7. The transportation cost per unit time from the vendor to the buyer is constant and independent of the quantity ordered. Thus, the total transportation cost per unit time is neglected.

2.1 Mathematical model

Investment in setup cost reduction

If I_S is an investment for the setup cost reduction, then it can be expressed as

$$\begin{aligned} I_S &= B \ln\left(\frac{S_0}{S}\right) \\ &= B(\ln S_0 - \ln S) \text{ for } 0 < S \leq S_0 \end{aligned}$$

where S_0 is the original setup cost, $B = \frac{1}{\delta}$, and $\delta =$ the percentage decrease in S per dollar increase in I_S.

The joint total expected cost per unit time for the vendor and the buyer can be expressed as

$$\begin{aligned} JATC(Q, k, S, L, m) &= ATC_b(Q, k, L) + ATC_v(Q, S, m) \\ &= \alpha B(\ln S_0 - \ln S) + \frac{D}{Q}\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right] \\ &+ \frac{Q}{2}\left[r_b C_b + r_v C_v \left\{m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}\right] + r_b C_b k\sigma\sqrt{L} \quad (1) \end{aligned}$$

For fixed positive integer m, the values of Q, $\Phi(k)$, and S are obtained as

$$Q = \left\{\frac{2D\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right]}{H(m)}\right\}^{\frac{1}{2}} \quad (2)$$

$$\Phi(k) = 1 - \frac{r_b C_b Q}{D\pi} \quad (3)$$

$$S = \frac{\alpha B Q m}{D} \quad (4)$$

For fixed Q, k, S, and m, the function $JATC(Q, k, L, S, m)$ is concave in L. Hence, for fixed Q, k, S, and m, the minimum value of $JATC(Q, k, L, S, m)$ is attained at the end points of the interval $[L_i, L_{i-1}]$.

Distribution free approach

We do not make any assumption for the distribution of the lead time demand X except that the cumulative distribution function (c.d.f.) F of the lead time demand belongs to the class $\Im$ of c.d.f. with mean DL and standard deviation $\sigma\sqrt{L}$.

The joint total cost for distribution free case is

$$\begin{aligned}
\text{Min} \quad JATC_f(Q, k, S, L, m) &= \alpha B(\ln S_0 - \ln S) + \frac{D}{Q}\Big[A + \frac{S}{m} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2} - k) \\
&+ C(L)\Big] + \frac{Q}{2}H(m) + r_b C_b k\sigma\sqrt{L} \qquad (5)
\end{aligned}$$

2.2 Conclusions

This study considered an integrated vendor-buyer supply chain model with the lead time, ordering quantity of the buyer, reorder point, quantity shifted from the vendor to the buyer, and the setup cost for the vendor as decision variables. An investment function was used to minimize the vendor's setup cost. The demand during lead time follows a normal distribution and in the second model, the distribution free approach is applied for the lead time demand. We minimized the joint total expected cost for the buyer and the vendor for both the normal distribution and the distribution free cases. Finally, we saved more amount of money compared to the previous studies related to this problem.

3 Manufacturing quality improvement and setup cost reduction in an integrated vendor-buyer supply chain system

This research improves the quality improvement of a single type of product and reduces vendor's setup cost in a single-vendor and single-buyer model. The buyer's demand is deterministic, but the lead time demand follows firstly a normal distribution and then follows no specific distribution except known mean and standard deviation. Based on the nature of lead time demand distribution, this research considers two different models. The procedure of reducing the vendor's setup cost and the manufacturing quality improvement of products are established analytically.

Assumptions

The assumptions remains same as previous model since this model is an extension of the previous model.

3.1 Mathematical model

If I_S is the investment for setup cost reduction, then it can be expressed as
$I_S = B\ln(\frac{S_0}{S})$ for $0 < S \leq S_0$, i.e., $I_S = B(\ln S_0 - \ln S)$, where S_0 is the initial setup cost, $B = \frac{1}{\delta}$, and $\delta =$ The percentage decrease in S per dollar increase in I_S.

Investment in quality improvement of the product

We assume the capital investment as I_θ for the reduction of the *out-of-control* probability θ. Thus, I_θ can be expressed as
$I_\theta = b\ln(\frac{\theta_0}{\theta})$ for $0 < \theta \leq \theta_0$, i.e., $I_\theta = b(\ln\theta_0 - \ln\theta)$, where θ_0 is the initial probability for which the production process can go *out-of-control* and $b = \frac{1}{\Delta}$, where Δ represents the

percentage decrease in θ per dollar increase in I_θ.

The joint total expected cost of the buyer and the vendor is

$$\begin{aligned} JATC(Q,k,S,\theta,L,m) &= ATC_b(Q,k,L) + ATC_v(Q,S,m) \\ &= \alpha\left[B(\ln S_0 - \ln S) + b(\ln\theta_0 - \ln\theta)\right] \\ &+ \frac{D}{Q}\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right] \\ &+ \frac{Q}{2}\left[r_bC_b + r_vC_v\left\{m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}\right] \\ &+ r_bC_bk\sigma\sqrt{L} + \frac{sDmQ\theta}{2} \end{aligned} \tag{6}$$

The minimum value of $JATC(Q,k,S,\theta,L,m)$ attends at the end point of the interval $[L_i, L_{i-1}]$. Now for fixed positive integer m, the values of Q, $\Phi(k)$, S, and θ are obtained from the following equations

$$Q = \left\{\frac{2D\left[A + \frac{S}{m} + \pi\sigma\sqrt{L}\psi(k) + C(L)\right]}{H(m) + sDm\theta}\right\}^{\frac{1}{2}} \tag{7}$$

$$\Phi(k) = 1 - \frac{r_bC_bQ}{D\pi} \tag{8}$$

$$S = \frac{\alpha BQm}{D} \tag{9}$$

$$\theta = \frac{2\alpha b}{sDmQ} \tag{10}$$

The optimal value of m can be obtained when

$$JATC(m^*-1) \geq JATC(m^*) \leq JATC(m^*+1)$$

Distribution free approach

We consider any distribution function (d.f.) F for the lead time demand in the class G of d.f.'s with mean DL and standard deviation $\sigma\sqrt{L}$. The model,in distribution free case, can be written as

$$
\begin{aligned}
\text{Min} \quad JATC_f(Q,k,S,\theta,L,m) &= \alpha\left[B(\ln S_0 - \ln S) + b(\ln\theta_0 - \ln\theta)\right] \\
&+ \frac{D}{Q}\Big[A + \frac{S}{m} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2} - k) \\
&+ C(L)\Big] + \frac{Q}{2}H(m) + r_b C_b k\sigma\sqrt{L} + \frac{sDmQ\theta}{2} \\
\text{subject to} \quad & 0 < S \leq S_0 \\
& 0 < \theta \leq \theta_0 \qquad (11) \\
\text{where} \quad H(m) &= r_b C_b + r_v C_v\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right]
\end{aligned}
$$

3.2 Conclusions

This research model considered an single-vendor single-buyer supply chain model with controllable lead time. This research improved the manufacturing quality of products and reduced the vendor's setup cost by using an investment function. The total system cost is reduced for the variable setup cost rather than fixed setup cost. The initial *out-of-control* probability was also reduced after using the investment function for quality improvement. This study suggests the managers of an industry to pay funds for collecting market information. The model can be extended by assuming the production of multi-item.

4 Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction

For quality improvement purposes often times, a manufacturing unit has to change certain parts of equipment. Any such changes in the assembly line manufacturing system or production process involves a cost known as the setup cost. Minimizing the setup cost and improving the product quality is of prime importance in today's competitive business arena. This research develops the effects of setup cost reduction and quality improvement in a two-echelon supply chain model with deterioration. The objective is to minimize the total cost of the entire supply chain model (SCM) by simultaneously optimizing setup cost, process quality, number of deliveries, and lot size.

Assumptions

We relax the common assumptions which were already used in the previous models.

1. As SSMD policy is used to save the holding cost of buyer, thus buyer pays transportation costs. For SSMD policy, it is assumed that there are some constant transportation costs and some variable costs. These both constant and variable transportation cost are paid by buyer.

2. Information regarding the inventory position and demand of the buyer are given to the supplier.

3. The vendor uses autonomation policy (automatically detects the defective item by machine, no human inspector is needed to inspect the defectiveness of items) to detect the imperfect production. As a result, if the system moves to *out-of-control* state from *in-control* state, it will continue production of defective items until the whole lot is produced.

4. A constant rate of deterioration is considered for products.

4.1 Mathematical model

Let y be the number of deteriorating units for the supplier. y can be expressed as $y = dA_s$. $y + dqT/2$ denotes the total number of deteriorating items for the entire SCM. With the following expressions $Q = Nq + y$ and $t_1 = \frac{Q}{P}$ and considering the initial and the total inventory for the entire SCM, we obtain

$$y + \frac{dqT}{2} = \frac{dT}{2P}\{2Dq + (Nq+y)(P-D)\}.$$

Hence,

$$\begin{aligned} A_s &= \frac{y}{d} \\ &= qT\left(\frac{D}{P} + \frac{N-1}{2} - \frac{DN}{2P}\right). \end{aligned} \tag{12}$$

Investment to reduce the *out-of-control* probability θ is given as

$$I_\theta(\theta) = b\ln\left(\frac{\theta_0}{\theta}\right) \qquad \text{for} \qquad 0 < \theta \leq \theta_0$$

Now $I_C(C)$, the investment for setup cost reduction is expressed as

$$I_C(C) = B\ln\left(\frac{C_0}{C}\right) \qquad \text{for} \qquad 0 < C \leq C_0$$

The expected annual defective cost is $\frac{sD\theta}{2}\left[Nq + \frac{2Ndq^2}{2D+dq}\left(\frac{D}{P} + \frac{N-1}{2} - \frac{DN}{2P}\right)\right]$. The integrated inventory cost for the entire SCM is

$$\begin{aligned} TC(\theta, N, q, C) &= \Big(\frac{D}{Nq} + \frac{d}{2N}\Big)(A + C + NF + VNq) + \frac{q}{2}\Big[(H_B + C_d d) \\ &+ (H_s + C_d d)\Big(\frac{(2-N)D}{P} + N - 1\Big)\Big] + \alpha\Big(G - b\ln\theta - B\ln C\Big) \\ &+ sD\frac{\theta}{2}\left[Nq + \frac{2Ndq^2}{2D+dq}\left(\frac{D}{P} + \frac{N-1}{2} - \frac{DN}{2P}\right)\right] \end{aligned} \tag{13}$$

for $0 < \theta \leq \theta_0$ and $0 < C \leq C_0$, α being the fractional cost of capital investment (e.g., the rate of interest).

We obtain the the values of the decision variables as

$$\theta = \frac{2b\alpha}{sD\left[Nq + \frac{2Ndq^2}{2D+dq}\left(\frac{D}{P} + \frac{N-1}{2} - \frac{DN}{2P}\right)\right]}, \tag{14}$$

$$N = \sqrt{\frac{\phi_2 - N^3\phi_3}{\phi_1}}, \tag{15}$$

where,

$$\begin{aligned} \phi_1 &= \frac{q}{2}\left[(H_s + C_d d)\left(1 - \frac{D}{P}\right)\right] + \frac{sD^2\theta q}{2D+dq}(1 + \frac{dq}{P}) \\ \phi_2 &= \frac{(2D+dq)(A+C)}{2q} \\ \phi_3 &= \frac{3}{2}\left(1 - \frac{D}{P}\right)\left(\frac{sD\theta dq^2}{2D+dq}\right) \end{aligned}$$

$$q = \sqrt{\frac{\rho_1}{\rho_4 + N\rho_2\{1 + \frac{2dq\rho_3(4D+dq)}{(2D+dq)^2}\}}}, \tag{16}$$

where,

$$\begin{aligned} \rho_1 &= \frac{D}{N}(A + C + NF) \\ \rho_2 &= \frac{sD\theta}{2} \\ \rho_3 &= \left(\frac{D}{P} + \frac{N-1}{2} - \frac{DN}{2P}\right) \\ \rho_4 &= \frac{1}{2}\left[(H_B + C_d d) + (H_s + C_d d)\{\frac{(2-N)D}{P} + N - 1\}\right] + \frac{dV}{2} \end{aligned}$$

$$C = \frac{2\alpha BNq}{2D+dq}. \tag{17}$$

4.2 Conclusions

The objective of this research was to minimize the total cost of the entire SCM while simultaneously optimizing lot size, number of deliveries, setup cost, and process quality. Two logarithmic investment functions for quality improvement and setup cost reduction, respectively were incorporated in this model. Quality improvement and setup cost reduction played a very significant role in improving efficiency of businesses and organizations from every sphere by reducing redundancy in costs and enhancing productivity thereby accounting for the flexibility of today's diverse business environment. Any adverse event would have a direct consequence on the business and customers leading to wastage of time and resource. An accurate expertise on the approaches of industries and organizations to implement these changes for a sustainable quality improvement is therefore critical. This model proved the global optimization solution of the decision variables.

5 Joint effect of price and demand on decision making in a supply chain management

This research model deals with a manufacturer-retailer supply chain model with decentralized decisions. Manufacturer's profit depends on the decisions made by retailer. Depending on the nature of the purchasing cost of retailer, this study considers two cases. In first case, retailer's purchase cost fully depends on the decisions made by retailer and in second case manufacturer determines the purchase cost of the retailer independently. Single-setup single-delivery (SSSD) and single-setup multi-delivery (SSMD) policies are considered for first and second cases, respectively. Retailer obtains the optimum selling price of product to maximize its profit. The customer's demand is price-sensitive whereas the lead time demand is considered as stochastic. The lead time demand follows a normal distribution. The distribution free approach is considered for known mean and standard deviation.

Assumptions

We relax the common assumptions which were already used in the previous models.

1. Due to the economic background of the people of suburban areas, increasing selling price is a important factor of decreasing demand. Thus, we assume that demand is dependent on selling price of retailer with the relation $D(p) = a - bp - cp^2$; $a, b, c > 0$.

2. Continuous review policy is considered i.e., retailer places an order when the inventory level reaches to the reorder point.

5.1 Mathematical model

Total expected profit for retailer is

$$\begin{aligned} TEP_b(Q,r,L,p) &= (p-C_b)D(p) - \frac{A(a-2bp-3cp^2)}{Q} - r_bC_b\left(\frac{Q}{2}+r-D(p)L\right) \\ &- \frac{\pi D(p)}{Q}\sigma\sqrt{L}\psi(k) - \frac{D(p)C(L)}{Q} \qquad (18) \\ &\psi(k) = \phi(k) - k[1-\Phi(k)] \text{ and} \\ &\phi(k) = \text{standard normal probability density function} \\ &\Phi(k) = \text{cumulative density function of normal distribution} \end{aligned}$$

Taking partial derivatives of the above equation with respect to Q and k, and equating to zero, we obtain

$$Q_b = \left\{\frac{2D(p)[A+\pi\sigma\sqrt{L}\psi(k)+C(L)]}{r_bC_b}\right\}^{1/2} \qquad (19)$$

$$\Phi(k) = 1 - \frac{r_bC_bQ_b}{\pi D(p)} \qquad (20)$$

The second order partial derivative of (18) with respect to L is negative, thus, the optimum value of L can be obtained at the end point of the interval $[L_i, L_{i-1}]$.
The optimal selling price are

$$p_1^* = \frac{\sqrt{B_1^2-4A_1C_1}-B_1}{2A_1} \qquad (21)$$

$$p_2^* = \frac{-\sqrt{B_1^2-4A_1C_1}-B_1}{2A_1} \qquad (22)$$

$$\begin{aligned} \text{where } A_1 &= -3c \\ B_1 &= 2\left\{cC_b - b + \frac{\alpha c}{Q}\right\} \\ C_1 &= C_bb + \frac{b\alpha}{Q} + a \qquad (23) \\ \alpha &= A + \pi\sigma\sqrt{L}\psi(k) + C(L) \end{aligned}$$

SSSD policy

In this case, manufacturer's total profit equation is

$$\begin{aligned} TEP_v(p, Q) &= \text{Total revenue} - \text{Setup cost} - \text{Holding cost} - \text{Material cost} \\ &= C_b D(p) - \frac{SD(p)}{Q} - r_v C_v \frac{QD(p)}{2P} - C_v D(p) \end{aligned} \tag{24}$$

The optimal lot size for manufacturer is

$$Q_v = \left\{ \frac{2SP}{r_v C_v} \right\}^{1/2} \tag{25}$$

Clearly, the order quantity for retailer must be equal to the lot size produced by manufacturer. The purchase cost for retailer is

$$C_b^* = \frac{r_v C_v D(p) \alpha}{r_b P S} \tag{26}$$

The manufacturer's total profit can be obtained by substituting values of the decision variables of the retailer's in the manufacturer's profit function i.e.,

$$TEP_v(Q_b^*, C_b^*, p^*) = C_b^* D(p^*) - \frac{SD(p^*)}{Q^*} - \frac{r_v C_v Q^* D(p^*)}{2P} - C_v D(p^*) \tag{27}$$

SSMD policy

The total profit equation for manufacturer is

$$\begin{aligned} TEP_v(Q, C_b, p, m) &= C_b D(p) - \frac{SD(p)}{mQ} - \frac{r_v C_v}{2} \Big[m \left(1 - \frac{D(p)}{P} \right) - 1 \\ &+ \frac{2D(p)}{P} \Big] Q - C_v D(p) \end{aligned} \tag{28}$$

Substituting the values of decision variables of retailer in manufacturer's total profit equation for SSMD policy, the maximum profit for the manufacturer can be obtained when the following inequality holds

$$TEP_v(Q_b^*, C_b^*, p^*, m-1) \leq TEP_v(Q_b^*, C_b^*, p^*, m) \geq TEP_v(Q_b^*, C_b^*, p^*, m+1)$$

Distribution free approach

We consider a distribution function (d.f) F for the lead time demand in the class G (say) of d.f.'s with the finite mean $D(p)L$ and standard deviation $\sigma\sqrt{L}$.

Total profit equation for retailer for distribution free case is

$$\begin{aligned} TEP_b^f(Q,k,p,L) &= (p-C_b)D(p) - \Big[\frac{AD(p)}{Q} + r_bC_b\left(\frac{Q}{2}+r-D(p)L\right) + C_bD(p) \\ &+ \frac{\pi D(p)}{Q}\sigma\sqrt{L}(\sqrt{1+k^2}-k)/2 + \frac{D(p)C(L)}{Q}\Big] \end{aligned} \tag{29}$$

The optimal selling price for distribution free case are

$$p_1^{f*} = \frac{\sqrt{B_2^2-4A_2C_2}-B_2}{2A_2} \tag{30}$$

$$p_2^{f*} = \frac{-\sqrt{B_2^2-4A_2C_2}-B_2}{2A_2} \tag{31}$$

Now, for SSSD policy the purchase cost for the retailer and the manufacturer's profit equation can be calculated as

$$C_b^{f*} = \frac{r_vC_vD(p)\beta}{r_bPS} \tag{32}$$

$$\begin{aligned} \text{and } TEP_v(Q_b^{f*},C_b^{f*},p^{f*}) &= C_b^{f*}D(p^{f*}) - \frac{SD(p^{f*})}{Q^{f*}} \\ &- \frac{r_vC_vQ^{f*}D(p^{f*})}{2P} - C_vD(p^{f*}) \end{aligned} \tag{33}$$

Similarly, for SSMD policy the total profit for manufacturer is

$$\begin{aligned} TEP_v(Q^{f*},C_b^{f*},p^{f*},m) &= C_b^{f*}D(p^{f*}) - \frac{SD(p^{f*})}{mQ^{f*}} \\ &- \frac{r_vC_v}{2}\left[m\left(1-\frac{D(p^{f*})}{P}\right)-1+\frac{2D(p^{f*})}{P}\right]Q^{f*} \\ &- C_vD(p^{f*}) \end{aligned} \tag{34}$$

The optimal profit for the manufacturer can be obtained as

$$TEP_v(Q_b^{f*},C_b^{f*},p^{f*},m-1) \le TEP_v(Q_b^{f*},C_b^{f*},p^{f*},m) \ge TEP_v(Q_b^{f*},C_b^{f*},p^{f*},m+1)$$

5.2 Conclusions

When manufacturer determined the purchasing cost for retailer by SSSD policy, it was found that both manufacturer and retailer were gainer. If the manufacturer determined retailer's purchasing cost of its own without taking retailer's decisions into consideration, then either manufacturer or retailer or both of them would face low profit. In that case, manufacturer fixed purchasing cost such that the difference between the total profit of manufacturer and retailer would be minimum. Otherwise, uncontrolled increase of purchasing cost might result increase of selling price of retailer which decreased the annual demand and hence the revenue also. Thus, the retailer might face a significant loss. Furthermore, for fixed purchase cost, the manufacturer can also follow SSMD policy to increase its profit.

6 Distribution free newsvendor model with consignment policy and retailer's royalty reduction

This study deals with a single period newsvendor problem with a consignment policy. A consignment policy is an agreement between any two parties called the consignor and the consignee. Both parties carry some portions of the holding cost, instead of just one. A new policy for paying the fixed fee to the consignee is introduced. This research considers no specific probability distribution for the customer's demand except mean and standard deviation. The solution of this model is obtained by using a distribution free approach. A comparison between the traditional supply chain policy and the consignment policy is also established. The price sensitivity for the mean demand is analyzed.

Assumptions

1. A single period newsvendor model is considered and no specific probability distribution is considered for the customer's demand. However, the mean and standard deviation of the demand are known.

2. In the traditional policy, total inventory carrying cost is carried by the retailer while in the consignment policy, the financial and the operational portion of the inventory holding cost are given by the manufacturer and the retailer, respectively.

3. The manufacturer pays a commission to the retailer per unit item sold, as well as a fixed fee.

4. During stockout, for each item, the manufacturer or retailer has to face a goodwill loss.

6.1 Mathematical model

Traditional policy

The Retailer's expected profit can be written as

$$\begin{aligned} E(\pi_r^{TS}) &= p(\mu+Q) - wQ - h_r^{TS}\left\{\frac{[\sigma^2+(Q-\mu)^2]^{1/2}-(\mu-Q)}{2}\right\} \\ &- s_r\left\{\frac{[\sigma^2+(Q-\mu)^2]^{1/2}-(Q-\mu)}{2}\right\} \\ &= p(\mu+Q) - wQ - \frac{h_r^{TS}+s_r}{2}[\sigma^2+(Q-\mu)^2]^{1/2} \\ &- \frac{h_r^{TS}-s_r}{2}(Q-\mu). \end{aligned} \tag{35}$$

The revenue of the manufacturer is wQ and the cost incurred by the manufacturer is the manufacturing cost. The expected profit of the manufacturer is

$$E(\pi_m^{TS}) = wQ - cQ = (w-c)Q. \tag{36}$$

In order to maximize the expected total profit of the retailer, we take the derivative of (35) with respect to Q.

$$\frac{\partial E(\pi_r^{TS})}{\partial Q} = p - w - \frac{h_r^{TS}+s_r}{2}[\sigma^2+(Q-\mu)^2]^{-1/2}(Q-\mu).$$

Equating the above equation to zero we obtain

$$Q_r^* = \mu + \frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}} \text{ where } \Gamma = \frac{2(p-w)-(h_r^{TS}-s_r)}{h_r^{TS}+s_r}. \tag{37}$$

Using (37), (36) can be written as

$$E(\pi_m^{TS}) = (w-c)Q_r^* = (w-c)\left[\mu + \frac{\sigma\Gamma}{\sqrt{1-\Gamma^2}}\right]. \tag{38}$$

Consignment policy

The expected total profit of the retailer for the consignment policy is

$$\begin{aligned} E(\pi_r^{CP}) &= \alpha(\mu + Q) - h_r^{CP} \left\{ \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (\mu - Q)}{2} \right\} \\ &- s_r \left\{ \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q - \mu)}{2} \right\} + A \\ &= \alpha(\mu + Q) - \frac{h_r^{CP} + s_r}{2} \sqrt{\sigma^2 + (Q-\mu)^2} \\ &+ \frac{h_r^{CP} - s_r}{2} (\mu - Q) + A. \end{aligned} \tag{39}$$

In order to maximize the profit of the retailer, we take the derivative of (39) with respect to Q which gives

$$\frac{\partial E(\pi_r^{CP})}{\partial Q} = \alpha - \frac{(h_r^{CP} + s_r)(Q - \mu)}{2\sqrt{\sigma^2 + (Q-\mu)^2}} - \frac{h_r^{CP} - s_r}{2}.$$

Now, equating the above equation to zero, we obtain

$$Q_r^{CP*} = \mu + \frac{\sigma \Gamma_{CP}}{\sqrt{1 - \Gamma_{CP}^2}} \text{ where } \Gamma = \frac{2\alpha - (h_r^{CP} - s_r)}{h_r^{CP} + s_r}. \tag{40}$$

The expected total profit of the manufacturer is

$$\begin{aligned} E(\pi_m^{CP}) &= p(\mu + Q) - \alpha\mu - \alpha Q - cQ - h_m^{CP} \left\{ \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (\mu - Q)}{2} \right\} \\ &- s_m \left\{ \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q - \mu)}{2} \right\} - A \\ &= p(\mu + Q) - cQ - \alpha Q - \alpha\mu - \frac{h_m^{CP} + s_m}{2} \sqrt{\sigma^2 + (Q-\mu)^2} \\ &+ \frac{h_m^{CP} - s_m}{2} (\mu - Q) - A. \end{aligned} \tag{41}$$

The joint total expected profit for manufacturer and retailer under consignment policy is

$$
\begin{aligned}
E(\pi_J^{CP}) &= \alpha(\mu+Q) - \frac{h_r^{CP}+s_r}{2}\sqrt{\sigma^2+(Q-\mu)^2} + \frac{h_r^{CP}-s_r}{2}(\mu-Q) + A \\
&\quad + p(\mu+Q) - cQ - \alpha Q - \alpha\mu - \frac{h_m^{CP}+s_m}{2}\sqrt{\sigma^2+(Q-\mu)^2} \\
&\quad + \frac{h_m^{CP}-s_m}{2}(\mu-Q) - A \\
&= (\alpha+p)(\mu+Q) - cQ - \alpha Q - \alpha\mu \\
&\quad - \frac{(h_r^{CP}+h_m^{CP})+(s_r+s_m)}{2}\sqrt{\sigma^2+(Q-\mu)^2} \\
&\quad + \frac{(h_r^{CP}+h_m^{CP})-(s_r+s_m)}{2}(\mu-Q).
\end{aligned} \tag{42}
$$

In order to maximize the joint total expected profit we take the derivative of (42) with respect to Q and equating to zero we obtain

$$
Q_J^{CP*} = \mu + \frac{\sigma\lambda}{\sqrt{1-\lambda^2}} \text{ where } \lambda = \frac{2(p-c)-((h_r^{CP}+h_m^{CP})-(s_r+s_m))}{(h_r^{CP}+h_m^{CP})+(s_r+s_m)} \tag{43}
$$

$$
\tag{44}
$$

Evaluation of per unit commission

$$
\alpha = \frac{h_r-s_r}{2} + \frac{h_r+s_r}{2(H_{CP}+S_{CP})}(2(p-c)-(H_{CP}-S_{CP})), \tag{45}
$$

where $H_{CP} = (h_r^{CP}+h_m^{CP})$ and $S_{CP} = (s_r+s_m)$.

Evaluation of the fixed fee paid by the manufacturer to the retailer

$$
\begin{aligned}
A &\le p(\mu+Q_r^{TS}) - wQ_r^{TS} - \frac{h_r^{TS}+s_r}{2}\sqrt{\sigma^2+(Q_r^{TS}-\mu)^2} \\
&\quad - \frac{h_r^{TS}-s_r}{2}(Q_r^{TS}-\mu) - \alpha(\mu+Q_r^{CP}) + \frac{h_r^{CP}+s_r}{2}\sqrt{\sigma^2+(Q_r^{CP}-\mu)^2} \\
&\quad + \frac{h_r^{CP}-s_r}{2}(Q_r^{CP}-\mu).
\end{aligned} \tag{46}
$$

Proposed way to evaluate the fixed fee

$$A = E(\pi_r^{TS}) - E(\pi_r^{CP})$$

The value of A will be negative if

$$E(\pi_r^{CP}) > E(\pi_r^{TS}) \tag{47}$$

Now, we obtain the ratio

$$r = \left|\frac{E(\pi_r^{TS})}{E(\pi_r^{CP})}\right| \tag{48}$$

If (47) holds then $r < 1$ and

$$A_n = rA = \left|\frac{E(\pi_r^{TS})}{E(\pi_r^{CP})}\right| A < A \tag{49}$$

Thus, the royalty which is to be given by the retailer can be reduced in this way. The expected profit for the retailer and the manufacturer for this new fixed cost will be

$$\begin{aligned} E(\pi_{rn}^{CP}) &= \alpha(\mu + Q_r^{CP}) - h_r^{CP}\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (\mu - Q_r^{CP})}{2}\right\} \\ &- s_r\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (Q_r^{CP} - \mu)}{2}\right\} + A_n \\ &= \alpha(\mu + Q_r^{CP}) - \frac{h_r^{CP} + s_r}{2}\sqrt{\sigma^2 + (Q_r^{CP} - \mu)^2} \\ &+ \frac{h_r^{CP} - s_r}{2}(\mu - Q_r^{CP}) + A_n \text{ and} \end{aligned} \tag{50}$$

$$\begin{aligned} E(\pi_{mn}^{CP}) &= p(\mu + Q_r^{CP}) - \alpha\mu - \alpha Q_r^{CP} - cQ_r^{CP} - h_m^{CP}\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (\mu - Q_r^{CP})}{2}\right\} \\ &- s_m\left\{\frac{[\sigma^2 + (Q_r^{CP} - \mu)^2]^{1/2} - (Q_r^{CP} - \mu)}{2}\right\} - A_n \\ &= p(\mu + Q_r^{CP}) - cQ_r^{CP} - \alpha Q_r^{CP} - \alpha\mu - \frac{h_m^{CP} + s_m}{2}\sqrt{\sigma^2 + (Q_r^{CP} - \mu)^2} \\ &+ \frac{h_m^{CP} - s_m}{2}(\mu - Q_r^{CP}) - A_n, \text{ respectively.} \end{aligned} \tag{51}$$

6.2 Conclusions

The optimal decisions was obtained for both traditional and consignment policy. It was observed that the joint profit for the consignment policy is greater than that of the traditional policy. To reduce the royalty for the retailer to the manufacturer a new method was provided. By using the proposed method the royalty for the retailer was reduced without affecting the joint total expected profit for both parties. The price sensitivity on demand was also been examined which showed that increment of a fraction of price may result reduction of total expected profit.

7 A multi-retailer supply chain model with backorder and variable production cost

This model considers an integrated supply chain model where a single vendor manufactures goods in a batch production process and supplies to a set of buyers over multiple times. Instead of assuming a fixed production rate which is commonly used in literature, variable production rate is considered by the vendor and also the production cost of the vendor is treated as a function of production rate. The continuous review inventory policy is applied by the buyers to inspect the inventory level and a crashing cost is incurred by all buyers to reduce lead time. The lead time demand follows a normal distribution. The unsatisfied demand at the buyers end are partially backordered. A service level constraint is incorporated corresponding to each buyer. A model is formulated to minimize the joint expected cost of the vendor-buyers supply chain system.

Assumptions

1. A single vendor supplies products to a number of buyers.
2. To satisfy the demand of each buyer, vendor supplies a total of Q quantity such that $Q = \sum_{i=1}^{n} q_i$.
3. The vendor manufactures mQ quantity against the order of q_i quantity of buyer i but the shipment should be in quantity Q over m times. The shipment procedure follows the relation $q_i = d_i \frac{Q}{D}$ i.e., $\frac{q_i}{d_i} = \frac{Q}{D}$.
4. Production rate is a variable quantity which varies within the range P_{min} ($P_{min} > D = \sum_{i=1}^{n} d_i$) and P_{max}.
5. The unit production cost of the vendor is a function of P.
6. Partial backorder is considered with backorder ratio β_i for ith retailer.

7. For ith retailer, we assume $L_{i,0} \equiv \sum_{j=1}^{n_i} b_{i,j}$. $L_{i,r}$ is the length of lead time with components $1, 2, ..., r$ crashed to their minimum duration. Thus, $L_{i,r}$ can be expressed as $L_{i,r} = L_{i,0} - \sum_{j=1}^{r}(b_{i,j} - a_{i,j})$, $r = 1, 2, ..., n$; and the lead time crashing cost per cycle $C_i(L_i)$ is expressed as $C_i(L_i) = c_{i,r}(L_{i,r-1} - L_i) + \sum_{j=1}^{r-1} c_{i,j}(b_{i,j} - a_{i,j})$, $L \in [L_{i,r}, L_{i,r-1}]$.

8. The lead time crashing cost entirely belongs to the buyer's cost component.

7.1 Mathematical model

The joint total expected cost for both vendor and the buyers ($JTEC$) is obtained from the following equation.

$$\begin{aligned} JTEC(Q, k_i, L_i, P, m) &= \sum_{i=1}^{n} \frac{D}{Q}\left[A_{bi} + \{\pi_i + \pi_{oi}(1-\beta_i)\}\sigma_i\sqrt{L_i}\psi(k_i) + \frac{A_v}{m} + R(L_i)\right] \\ &+ \sum_{i=1}^{n} h_{bi}\left[\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i} + (1-\beta_i)\sigma_i\sqrt{L_i}\psi(k_i)\right] \\ &+ \frac{Q}{2}h_v\left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + D\left(\frac{a_1}{P} + a_2 P\right) \end{aligned} \tag{52}$$

We note that the second order partial derivative of the joint total cost function with respect to L_i is

$$\begin{aligned} \frac{\partial^2 JTEC(Q, k_i, L_i, P, m)}{\partial L_i^2} &= -\frac{D}{4Q}\{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\psi(k_i)L_i^{-3/2} \\ &- (k_i\sigma_i + (1-\beta_i)\sigma_i\psi(k_i))\frac{h_{bi}L_i^{-3/2}}{4} \end{aligned} \tag{53}$$

Which is a negative term for $0 < \beta_i < 1$ and all the positive values of the parameters and decision variables present in (53). Therefore, for fixed Q, k_i, P, and m, the function $JTEC(Q, k_i, L_i, P, m)$ is concave in L_i. Thus, for fixed Q, k_i, P, and m, the minimum value of $JTEC(Q, k_i, L_i, P, m)$ attains at the end point of the interval $[L_{i,j}, L_{i,j-1}]$. Now for fixed positive integer m, and for any fixed value of L_i the values of Q, $\Phi(k_i)$, and P

can be obtained as,

$$Q = \left\{ \frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + [\pi_i + \pi_{0i}(1-\beta_i)]\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n} \frac{h_{bi}}{D} d_i + h_v \left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right]} \right\}^{1/2} \tag{54}$$

$$\Phi(k_i) = 1 - \frac{h_{bi}}{\frac{D}{Q}(\pi_i + \pi_{0i}(1-\beta_i)) + (1-\beta_i)} \tag{55}$$

$$P = \left\{ \frac{2a_1 - Qh_v(m-2)}{2a_2} \right\}^{1/2} \tag{56}$$

7.2 Conclusions

This study proposed a single vendor and multiple buyers supply chain model. Variable lead time was considered at the buyer's end. The lead time demand was assumed to follow a normal distribution. The vendor's production rate was considered as variable rather than as a fixed entity. Moreover, the unit production cost was also treated as a variable that was dependent on the production rate, and a special type of function was considered to establish the relation between the production rate and the unit production cost. At the end of the production, the finished goods were delivered to a number of buyers through a multi-delivery policy.

8 Relation between quality of products and production-rate in a single-vendor multi-retailer joint economic lot size model with variable production cost

This study deals with an integrated single vendor multi-buyer supply chain model with variable production rate and imperfect quality. The production rate of the vendor is treated as flexible. The quality of the products is also dependent on the production rate. The relation between process quality and production rate is established in this context. Moreover, the unit production cost is also considered to be a function of the production rate. End products are delivered to satisfy the demands of buyers over multiple time segments. The lead time is variable and a lead time crashing cost is incorporated by the buyers to lower the lead time, whereas the lead time demand is considered to be stochastic and to follow a normal distribution. The aim of this research is to analyze how the flexibility of the production rate affects the entire supply chain cost under a single-setup multi-delivery policy.

Assumptions

We relax the common assumptions which were already used in the previous models.

1. The unit production cost is dependent on the production rate P and the quality of the product deteriorates with increasing production rate.

2. The elapsed time after the production system goes *out-of-control* is an exponentially distributed random variable and the mean of the exponential distribution is a decreasing function of the production rate [21].

8.1 Mathematical model

The objective of this research is to obtain centralized decisions for both the vendor and the buyers to minimize the joint total supply chain cost. The joint total expected cost for both the vendor and the buyers ($JTEC$) can be expressed as

$$
\begin{aligned}
JTEC(Q, k_i, L_i, P, m) &= \sum_{i=1}^{n} \Big[\frac{A_{bi} D}{Q} + h_{bi} \left\{ \frac{Q}{2D} d_i + k_i \sigma_i \sqrt{L_i} \right\} \\
&+ \frac{D}{Q} \pi_i \sigma_i \sqrt{L_i} \psi(k_i) + R(L_i) \frac{D}{Q} \Big] \\
&+ \frac{A_v D}{mQ} + \frac{Q}{2} h_v \left[m \left(1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] \\
&+ RD\alpha f(P) \frac{Q}{2P} + DC(P). \qquad (57)
\end{aligned}
$$

When the machines are inoperative i.e., the production process ceases, there is no chance of any defective products to be created or the probability of the process going *out-of-control* is zero. As the machines change into operation mode, the chances of the arrival of defective goods appear. We consider an increasing function $f(P)$ of production rate P such that the mean time to failure $\frac{1}{f(P)}$ becomes a decreasing function of P. We introduce three different cases with three different functions to define the mean time to failure.

$$\text{Case 1: } \frac{1}{f(P)} = \frac{1}{b_1 P} \text{ (The quality function } f(P) \text{ is linear in } P), \qquad (58)$$

$$\text{Case 2: } \frac{1}{f(P)} = \frac{1}{b_2 P + c_2 P^2} \text{ (The quality function } f(P) \text{ is quadratic in } P), \qquad (59)$$

$$\text{Case 3: } \frac{1}{f(P)} = \frac{1}{b_3 P + c_3 P^2 + d_3 P^3} \text{ (The quality function } f(P) \text{ is cubic in } P). \qquad (60)$$

where $b_1,\ b_2,\ c_2,\ b_3,\ c_3$ and d_3 are non-negative real numbers that provide the best fit for the function $f(P)$ as well as $\frac{1}{f(P)}$. We denote Q_p, k_i^p, and P_p for Case p, $p = 1, 2,$ and 3, denote the three cases described above, respectively.

Case 1 (Linear quality function)

$$Q_1 = \left[\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i \sigma_i \sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n} \frac{h_{bi}}{D} d_i + h_v \left[m\left(1 - \frac{D}{P_1}\right) - 1 + \frac{2D}{P_1}\right] + \frac{RD\alpha b_1 P_1}{P_1}} \right]^{1/2}, \tag{61}$$

$$\Phi(k_i^1) = 1 - \frac{h_{bi} Q_1}{D\pi_i}, \tag{62}$$

$$P_1 = \left[\frac{2a_1 D - Q_1 h_v D(m-2)}{2Da_2} \right]^{1/2}, \tag{63}$$

$$\begin{aligned} JTEC_1(Q_1, k_i, L_i, P_1, m) &= \sum_{i=1}^{n} \Big[\frac{A_{bi} D}{Q_1} + h_{bi} \left\{ \frac{Q_1}{2D} d_i + k_i \sigma_i \sqrt{L_i} \right\} \\ &+ \frac{D}{Q_1} \pi_i \sigma_i \sqrt{L_i} \psi(k_i) + R(L_i) \frac{D}{Q_1} \Big] \\ &+ \frac{A_v D}{m Q_1} + \frac{Q_1}{2} h_v \left[m\left(1 - \frac{D}{P_1}\right) - 1 + \frac{2D}{P_1} \right] \\ &+ RD\alpha b_1 P_1 \frac{Q_1}{2P_1} + D\left(\frac{a_1}{P_1} + a_2 P_1 \right). \end{aligned} \tag{64}$$

Case 2 (Quadratic quality function)

$$Q_2 = \left[\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i \sigma_i \sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n} \frac{h_{bi}}{D} d_i + h_v \left[m\left(1 - \frac{D}{P_2}\right) - 1 + \frac{2D}{P_2}\right] + \frac{RD\alpha(b_2 P_2 + c_2 P_2^2)}{P_2}} \right]^{1/2}, \tag{65}$$

$$\Phi(k_i^2) = 1 - \frac{h_{bi} Q_2}{D\pi_i}, \tag{66}$$

$$P_2 = \left[\frac{2a_1 D - Q_2 h_v D(m-2)}{2Da_2 + R\alpha D Q_2 b} \right]^{1/2}, \tag{67}$$

$$\begin{aligned} JTEC_2(Q_2, k_i, L_i, P_2, m) &= \sum_{i=1}^{n} \Big[\frac{A_{bi} D}{Q_2} + h_{bi} \left\{ \frac{Q_2}{2D} d_i + k_i \sigma_i \sqrt{L_i} \right\} \\ &+ \frac{D}{Q_2} \pi_i \sigma_i \sqrt{L_i} \psi(k_i) + R(L_i) \frac{D}{Q_2} \Big] \\ &+ \frac{A_v D}{m Q_2} + \frac{Q_2}{2} h_v \left[m\left(1 - \frac{D}{P_2}\right) - 1 + \frac{2D}{P_2} \right] \\ &+ RD\alpha(b_2 P_2 + c_2 P_2^2) \frac{Q_2}{2P_2} + D\left(\frac{a_1}{P_2} + a_2 P_2 \right). \end{aligned} \tag{68}$$

Case 3 (Cubic quality function)

$$Q_3 = \left[\frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + \pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n} \frac{h_{bi}}{D} d_i + h_v \left[m\left(1 - \frac{D}{P_3}\right) - 1 + \frac{2D}{P_3}\right] + \frac{RD\alpha(b_3P_3 + c_3P_3^2 + d_3P_3^3)}{P_3}} \right]^{1/2}, \qquad (69)$$

$$\Phi(k_i^3) = 1 - \frac{h_{bi}Q_3}{D\pi_i}, \qquad (70)$$

$$P_3 = \left[\frac{2Da_1 - Q_3h_vD(m-2)}{2(d_3R\alpha DQ_3P_3 + Da_2) + R\alpha DQ_3c_2} \right]^{1/2}, \qquad (71)$$

$$\begin{aligned} JTEC_3(Q_3, k_i, L_i, P_3, m) &= \sum_{i=1}^{n} \left[\frac{A_{bi}D}{Q_3} + h_{bi}\left\{ \frac{Q_3}{2D}d_i + k_i\sigma_i\sqrt{L_i} \right\} \right. \\ &+ \left. \frac{D}{Q_3}\pi_i\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q_3} \right] \\ &+ \frac{A_vD}{mQ_3} + \frac{Q_3}{2}h_v\left[m\left(1 - \frac{D}{P_3}\right) - 1 + \frac{2D}{P_3} \right] \\ &+ RD\alpha(b_3P_3 + c_3P_3^2 + d_3P_3^3)\frac{Q_3}{2P_3} + D\left(\frac{a_1}{P_3} + a_2P_3\right). \qquad (72) \end{aligned}$$

8.2 Conclusions

The effects of mean time to failure for the three cases stated above on the entire supply chain cost was examined, which provides a tremendous managerial insight for the industry. Again, the model was also studied when the mean time to failure was independent of the production rate. Moreover, the unit production cost was also treated as a variable that was dependent on the production rate, and a special type of function was considered to establish the relation between the production rate and the unit production cost. At the end of the production, the finished goods were delivered to a number of buyers through a multi-delivery policy.

9 A study on three different dimensional facility location problems

In supply chain strategy, designing a network is one of the most important part. This model deals with various dimensional facility location models. Initially, this study begins with two echelon facility location model of dimension two. Then, it is extended to three dimensional model by adding commodity type and then, different types of transportation modes are added to make it four dimensional model. Delivery lead time and outside suppliers are assumed to meet the retailer's demand too. This research compares the optimal solutions among every dimension. A study on the procedure of reducing the total cost of the supply chain network is also incorporated by applying a small change in constraint set.

Assumptions

1. The model deals with two echelon supply chain network with plants and warehouses having fixed capacities..
2. Outsider suppliers are considered to fulfill the demands of the retailers too.
3. An annual fixed cost is needed for each warehouse and plant to be opened.
4. Plant and warehouse at each site have a fixed inventory holding.

9.1 Mathematical model

Problem P1

Here, we assume a capacitated facility location problem in dimension two. The dimensions are considered as two locations in between which the commodity are to be shifted. This model deals with two echelon supply chain. i.e., the commodities are to be delivered from plants to warehouses and from warehouses to retailers.

Objective function:

$$
\begin{aligned}
Minf &= \sum_{i\in I}\sum_{j\in J} TC_{ij}x_{ij}D_i + \sum_{k\in K}\sum_{j\in J} PTC_{jk}y_{jk}WC_j \\
&+ \sum_{i\in I} OSC_i D_i s_i + \sum_{j\in J} IC_j I_j + \sum_{k\in K} JC_k J_k + \sum_{j\in J} TCW_j z_j \\
&+ \sum_{k\in K} TCP_k c_k + \sum_{i\in I}\sum_{j\in J} MD_i TWR_{ij}x_{ij} + \sum_{j\in J}\sum_{k\in K} MWC_j TPR_{jk}y_{jk}
\end{aligned}
$$

Subject to the constraints:

$$
\begin{aligned}
&\sum_{j\in J} x_{ij} \geq 1 \\
&s_i \geq 1 \\
&\sum_{k\in K} y_{jk} \geq 1 \\
&\sum_{i\in I} D_i x_{ij} + I_j \leq WC_j z_j \\
&\sum_{j\in J} WC_j y_{jk} + J_k \leq PC_k c_k \\
&z_j, c_k \in \{0,1\} \forall j \in J, k \in K \\
&0 \leq x_{ij}, y_{jk}, s_i \leq 1
\end{aligned}
$$

Problem P2

Now, product type is added as another new dimension.

Objective function:

$$
\begin{aligned}
Minf &= \sum_{i\in I}\sum_{j\in J}\sum_{p\in P} TC_{ijp}x_{ijp}D_{ip} + \sum_{k\in K}\sum_{j\in J}\sum_{p\in P} PTC_{jkp}y_{jkp}WC_j \\
&+ \sum_{i\in I}\sum_{p\in P} OSC_{ip}D_{ip}s_{ip} + \sum_{j\in J}\sum_{p\in P} IC_{jp}I_{jp} + \sum_{k\in K}\sum_{p\in P} JC_{kp}J_{kp} + \sum_{j\in J} TCW_j z_j \\
&+ \sum_{k\in K} TCP_k c_k + \sum_{i\in I}\sum_{j\in J}\sum_{p\in P} M_p D_{ip} TWR_{ijp}x_{ijp} + \sum_{j\in J}\sum_{k\in K}\sum_{p\in P} M_p WC_j TPR_{jkp}y_{jkp}
\end{aligned}
$$

Subject to the constraints:

$$\sum_{j \in J} x_{ijp} \geq 1$$
$$s_{ip} \geq 1$$
$$\sum_{k \in K} y_{jkp} \geq 1$$
$$\sum_{i \in I} \sum_{p \in P} D_{ip} x_{ijp} + \sum_{p} I_{jp} \leq WC_j z_j$$
$$\sum_{j \in J} \sum_{p \in P} WC_j y_{jkp} + \sum_{p} J_{kp} \leq PC_k c_k$$
$$z_j, c_k \in \{0,1\} \forall j \in J, k \in K$$
$$0 \leq x_{ijp}, y_{jkp}, s_{ip} \leq 1$$

Problem P3

Again, another dimension is added here. Now, the type of transportation mode is set as new additional dimension.

Objective function:

$$\begin{aligned} Minf \; = \; & \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} TC^t_{ijp} x^t_{ijp} D_{ip} + \sum_{t \in T} \sum_{k \in K} \sum_{j \in J} \sum_{p \in P} PTC^t_{jkp} y^t_{jkp} WC_j \\ + \; & \sum_{i \in I} \sum_{p \in P} \sum_{t \in T} OSC^t_{ip} D_{ip} s^t_{ip} + \sum_{j \in J} \sum_{p \in P} IC_{jp} I_{jp} + \sum_{k \in K} \sum_{p \in P} JC_{kp} J_{kp} \\ + \; & \sum_{j \in J} TCW_j z_j + \sum_{k \in K} TCP_k c_k + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} M_p D_{ip} TWR^t_{ijp} x^t_{ijp} \\ + \; & \sum_{t \in T} \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} M_p WC_j TPR^t_{jkp} y^t_{jkp} \end{aligned}$$

Subject to the constraints:

$$\sum_{t\in T}\sum_{j\in J} x^t_{ijp} \geq 1$$
$$\sum_{t\in T} s^t_{ip} \geq 1$$
$$\sum_{t\in T}\sum_{k\in K} y^t_{jkp} \geq 1$$
$$\sum_{t\in T}\sum_{i\in I}\sum_{p\in P} D_{ip}x^t_{ijp} + \sum_{p} I_{jp} \leq WC_j z_j$$
$$\sum_{t\in T}\sum_{j\in J}\sum_{p\in P} WC_j y^t_{jkp} + \sum_{p} J_{kp} \leq PC_k c_k$$
$$z_j, c_k \in \{0,1\} \forall j \in J, k \in K$$
$$0 \leq x^t_{ijp}, y^t_{jkp}, s^t_{ip} \leq 1$$

Change in constraint set

If the demand constraint sets for outside suppliers of the above three problems P1, P2 and P3 are changed from each retailer to for all retailers i.e., if the demand is divided into all retailers, then, the total cost will be minimized.

9.2 Conclusions

We concluded that the increment or reduction of cost depends on the type of the dimension used. Two separate type of dimensions were used such as product type and transportation mode. The types of products depend on the retailer's demand, hence, these create the increments of costs. Again, the mode of transportation is independent of the retailer's demand which indicates the reduction of the cost. Lastly, a small change in the constraint sets was considered which results the decrement of total cost.

Future extensions

There are many possible extensions of this research. Some of them are pointed below.

1. The single setup multi-delivery policy can be extended by unequal shipment.

2. The dimension of a facility location problem can be extended by generalized n dimension for $n \in N$ (Set of natural number).

3. A fruitful research can be done by assuming a discrete investment to reduce setup cost instead of continuous investment.

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