Synopsis of the Thesis

SOME DEVELOPMENTS OF SOFT COMPUTING METHODS FOR TSP UNDER UNCERTAIN OPTIMIZATION PARADIGMS

Submitted to the

VIDYASAGAR UNIVERSITY

BY

SAMIR MAITY
Registration No: 366/Ph.D./Sc. 03-04-2013
Department of Computer Science
Vidyasagar University
Midnapore-721102, INDIA

September, 2016

PUBLICATION

The list of Published Papers :

1. S. Maity, A. Roy, M. Maiti, *A Modified Genetic Algorithm for solving uncertain Constrained Solid Travelling Salesman Problems,* Computers & Industrial Engineering, (**Elsevier**) **83**(2015), 273-296, **SCI**, Impact Factor 2.028.
2. S. Maity, A. Roy, M. Maiti, *An imprecise Multi-Objective Genetic Algorithm for uncertain Constrained Multi-Objective Solid Travelling Salesman Problem*, Expert Systems With Applications, (**Elsevier**) **46**(2016), 196-223, **SCI**, Impact Factor 2.986
3. S. Maity, A. Roy, M. Maiti, *Constrained Solid Travelling Salesman Problem Solving by Rough GA Under Bi-Fuzzy Coefficients,* Advances in Intelligent Systems and Computing 404, (**Springer**) DOI 10.1007/978-81-322-2695-6-36.
4. M. Maiti, S. Maity, A. Roy, *An Improved Genetic Algorithm and Its Application in Constrained Solid TSP in Uncertain Environments*, Proceedings in Mathematics & Statistics 125, (**Springer**) DOI 10.1007/978-81-322-2301-6-14
5. A. Roy, S. Maity, M. Maiti, *Constrained solid travelling salesman problem using Adaptive Genetic Algorithm in uncertain environment*, **IEEE**, DOI:10.1109/ICIEV.2015.7334044

The list of Communicated Papers

1. Rough Genetic Algorithm for Constrained Solid TSP with Interval Valued Costs and Times.
   Communicated to International Journal of *Fuzzy Information and Engineering* (**Elsevier**)
2. A Rough Multi-Objective Genetic Algorithm for uncertain Constrained Multi-Objective Solid Travelling Salesman Problem.
   Communicated to International Journal of *Discrete Applied Mathematics* (**Elsevier**)
3. An Intelligent Hybrid Algorithm for four Dimensional TSP (4DTSP).
   Communicated to International Journal of *Industrial Information Integration* (**Elsevier**)
4. A new Evolutionary Hybrid Algorithm for restricted 4- Dimensional TSP (r-4DTSP) in Uncertain Environment. Communicated to International Journal of *Swarm and Evolutionary Computation* (**Elsevier**)

Introduction

Soft Computing (SC) is the fusion of methodologies that were designed to model and enable solutions to real world problems, which are otherwise too difficult to formulate, mathematically. SC is a consortium of methodologies that works synergistically and provides, in one form or another, flexible information processing capability for handling real-life ambiguous situations. Travelling salesman problem (TSP) is central to operations research and management science. It is now widely recognized that some of the most successful applications of operations research are encountered in TSP, most significantly in the airline industry where they underlay almost every aspect of strategic, tactical and operational planning. Still there have no state of art algorithm that exactly solve TSP in polynomial time. So in the present research, model different types of soft computing methods and develop the real world problem as solid travelling salesman problem under stochastic as well as non stochastic uncertainties. The proposed algorithms are tested for the efficiency by solving the standard problems also taking statistical tests. These algorithms can be used to solve discrete optimization problems.

Soft Computing and Uncertainties

The final aim is to develop a computer or a machine which will work in a similar way as human beings can do, i.e. the wisdom of human beings can be replicated in computers in some artificial manner. The motivation for such an extension is the expected decrease in computational load and consequent increase of computation speeds that permit more robust system [16]. SC has three main branches: fuzzy systems, evolutionary computation, artificial neural computing, with the latter subsuming machine learning (ML) and probabilistic reasoning (PR), belief networks, chaos theory, parts of learning theory and wisdom based expert system (WES), etc.

Optimization is a subject that attempts to find best possible solution for a problem. Optimization problems are quite common in computer science, whenever real-world applications are considered [35].

Uncertainty intrudes into plans for the future, interpretations of the past, and decision in the present. There are many kinds of uncertainty. In real world values are vague, fuzzy, confidence, ambiguity, inconsistent, incomplete, imprecise, general, anomalous, incongruent, ignorant and irrelevant. In the present study we used interval valued, fuzzy and rough with their different combinations as the various parameters in the optimization problem.

Main Components of Soft Computing

Fuzzy System

A fuzzy expert system consists of a fuzzy rule base, a fuzzification module, an inference engine, and a defuzzification module. The fuzzification module pre-processes the input values submitted to the fuzzy expert system.

Artificial Neural Networks

Artificial neural networks (ANN), or simply neural networks, can be loosely defined as large sets of interconnected simple units which execute in parallel to perform a common global task.

Fuzzy Logic

The human beings deal with imprecise and uncertain information as we go about our day to day routines. This can be gleaned from the language we use which contains many qualitative and subjective words and phrases such as quite expensive, very young, or a little far, expensive, etc. In human information processing, approximate reasoning is used and tried to accommodate varying degrees of imprecision and uncertainty in the concepts and tokens of information that we deal with in fuzzy logic.

Biologically Inspired Methods

Biologically inspired methods is a general term pertaining to computing which is inspired by nature. Over the last thirty years many differing strategies have been developed, ranging from Artificial Neural Networks, Evolutionary Computation, Fuzzy Sets to Ant Colony Optimization, Genetic Algorithm and Swarm Optimization, etc. These differing algorithms have been applied to a number of complex problems, such as: signal and image processing, data visualization, data mining, and combinatorial optimization.

Genetic Algorithm

The genetic algorithm is another machine learning technique which derives its behaviour from an evolutionary biology metaphor. Genetic algorithms were formalised by Holland [15] in 1975 as a model of adaptation. In simple genetic algorithms, by Goldberg [14] randomly generated solution strings are formed into a population. The strings are decoded and then evaluated according to a fitness/objective function. Following this, individuals are selected to undergo reproduction to produce offspring (individuals for the next generation).

Ant Colony Optimization

One of the first behaviors studied by entomologists was the ability of ants to find the shortest path between their nest and a food source. From these studies and observations followed the first algorithmic models of the foraging behavior of ants developed by Marco Dorigo [9].

Particle Swarm Optimization

The particle swarm optimization (PSO) algorithm is a population-based search algorithm based on the simulation of the social behavior of birds within a flock. The initial intent of the particle swarm concept was to graphically simulate the graceful and unpredictable choreography of a bird flock [21], with the aim of discovering patterns that govern the ability of birds to fly synchronously.

Hybrid Intelligent Systems

Traditional methods of optimization are not robust to dynamic changes in the environment and they require a complete restart for providing a solution. In contrary, evolutionary computation can be used to adapt solutions to changing circumstances. Hybridization of evolutionary algorithms is getting popular due to their capabilities in handling several real world problems involving complexity, noisy environment, imprecision, uncertainty and vagueness. Usually grouped under the term evolutionary computation or evolutionary algorithms, we find the domains of genetic algorithms [14], and genetic programming [22]. They all share a common conceptual base of simulating the evolution of individual structures via processes of selection, mutation, and reproduction. The merging of ACO, GA and PSO can be realized in different directions, resulting in systems with different characteristics given in this thesis.

Travelling Salesman Problems

The travelling salesman problem is stated as follows: given a number of cities with associated city to city distances, what is the shortest round trip tour that visits each city exactly once and returns to the start city [8]. The problem sounds quite simple, however as the number of cities in the problem increases so too does the number of permutations of valid tours e.g. for 5 cities 12, 7 cities 360 and for 9 cities 20160 possible permutations (for a 60 city problem it is possible that the number of permutations is of the same order of magnitude as the total number of atoms in the universe). Thus attempting to find the minimal distance tour in anything but very small problems is computationally expensive.

Different types of TSPs

Several types of TSP that are studied in the literature have been originated from various real life or potential applications. Let us first consider some of these variations that can be reformulated as a TSP using relatively simple transformations. These are TSPs with time windows [12], stochastic TSP [4], double TSP [32], asymmetric TSP [26, 27], TSP with precedence constraints [33, 28], etc.

Multi-dimensional TSPs

(a) Classical TSP(2DTSP)

In a classical two-dimensional TSP, a salesman has to travel N cities at minimum cost. In this tour, salesman starts from a city, visit all the cities exactly once and comes to the starting city using minimum cost. Let $c(i, j)$ be the cost for travelling from i-th city to j-th city. Then the problem can be mathematically formulated as:

$$\left.\begin{array}{r l}\text{Minimize} & Z = \sum_{i \neq j} c(i, j)x_{ij} \\ \text{subject to} & \sum_{i=1}^{N} x_{ij} = 1 \ \text{ for } j = 1, 2, ..., N \\ & \sum_{j=1}^{N} x_{ij} = 1 \ \text{ for } i = 1, 2, ..., N \\ & \sum_{i \in S}^{N} \sum_{j \in S}^{N} x_{ij} \leq |S| - 1, \forall S \subset Q \\ & \sum_{i=1}^{N} \sum_{j=1}^{N} t(i, j)x_{ij} \leq t_{max} \\ \textit{where} & x_{ij} \in \{0, 1\}, i, j = 1, 2.., N.. \end{array}\right\} \tag{1}$$

where x_{ij} is the decision variable and $x_{ij} = 1$ if the salesman travels from city-i to city-j, otherwise $x_{ij} = 0$. Then the above 2DTSP reduces to

$$\left.\begin{array}{l}\text{determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \\ \text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}) + c(x_N, x_1) \\ \qquad \textit{where} \ \ x_i \neq x_j, i, j = 1, 2..., N. \end{array}\right\} \tag{2}$$

along with sub tour elimination criteria

$$\left.\begin{array}{r l} & \sum_{i \in S}^{N} \sum_{j \in S}^{N} x_{ij} \leq |S| - 1, \forall S \subset Q \\ \textit{where} & x_{ij} \in \{0, 1\}, i, j = 1, 2.., N.. \end{array}\right\} \tag{3}$$

Later on, this constraint Equ. 3 is not mentioned explicitly in the formulation different TSP models, assuming that it is automatically satisfied for a feasible solution.

Proposed Solid TSP(3DTSP)

(a) Proposed Solid TSP(3DTSP)

In a Solid TSP, a salesman has to travel N cities by choosing any one of the P types of conveyances available using minimum cost. risk/discomfort factors in travelling from one city to another using different vehicles are different. The salesman should choice such a path and conveyances. Let $c(i, j, k)$ be the cost for travelling from i-th city to j-th city using k-th type conveyance. Then the salesman has to determine a complete tour (x_1, x_2, ...,x_N, x_1) and corresponding conveyance types (v_1, v_2, ..., v_P) to be used for the tour, where $x_i \in \{1, 2, ..N\}$ for $i = 1, 2, ..., N$, $v_i \in \{1, 2, ..P\}$ for $i = 1, 2, ..., N$ and all x_i are distinct. Then the problem can be mathematically formulated as:

Determine a complete tour $(x_1, x_2, ..., x_N, x_1)$ using any one available corresponding conveyance in each step from the vehicle types $(v_1, v_2, ..., v_P)$ so as

$$\left.\begin{array}{l}\text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_i) + c(x_N, x_1, v_l), \\ \textit{where} \ \ x_i \neq x_j, i, j = 1, 2...N, \quad v_i, v_l \in \{1, 2.., orP\} \end{array}\right\} \tag{4}$$

(b) Solid TSP with restricted conveyances (3DTSPwR)

In real life, it is seen that in all stations, all types of conveyances may not available due to

the geographical position of the station,weather conditions, etc. So it is more realistic, that restricted conveyances are available in different stations. Considering the availability of the conveyances, we design the STSP with restricted condition as below:
Let $c(i, j, k)$ be the cost for travelling from i-th city to j-th city using k-th type conveyance. Then the salesman has to determine a complete tour (x_1, x_2, ...,x_N, x_1) and corresponding conveyance types (v_1, v_2, ..., v_S) to be used for the tour, where $x_i \in \{1, 2, ..N\}$ for $i = 1, 2, ..., N$, $v_i \in \{1, 2, ..S\}$ for $i = 1, 2, ..., N$ and all x_is are distinct. Also $v_i \in \{1, 2, ..S\}$ provides maximum available S($\leq$ P) types of conveyances. Then the problem can be mathematically formulated as:

Determine a complete tour $(x_1, x_2, ..., x_N, x_1)$ using any one available corresponding conveyance in each step from the vehicle types $(v_1, v_2, ..., v_S)$ so as

$$\left.\begin{array}{c} \text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_i) + c(x_N, x_1, v_l), \\ where \;\; x_i \neq x_j, i, j = 1, 2...N, \quad v_i, v_l \in \{v_1, v_2.., v_S\} \end{array}\right\} \quad (5)$$

(c) STSP with risk/discomfort Constraints (CSTSP)

Let $c(i, j, k)$ be the cost for travelling from i-th city to j-th city using k-th type conveyance and $r(i, j, k)$ be the risk/discomfort factor in travelling from i-th city to j-th using k-th type conveyances. Then the salesman has to determine a complete tour (x_1, x_2, ...,x_N, x_1) and corresponding conveyance types (v_1, v_2, ..., v_P) to be used for the tour, where $x_i \in \{1, 2, ..N\}$ for $i = 1, 2, ..., N$, $v_i \in \{1, 2, ..P\}$ for $i = 1, 2, ..., N$ and all x_i are distinct. Then the problem can be mathematically formulated as:

Determine a complete tour $(x_1, x_2, ..., x_N, x_1)$ using any one available corresponding conveyance in each step from the vehicle types $(v_1, v_2, ..., v_P)$ so as

$$\left.\begin{array}{c} \text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_i) + c(x_N, x_1, v_l), \\ \text{subject to} \quad \sum_{i=1}^{N-1} r(x_i, x_{i+1}, v_i) + r(x_N, x_1, v_l) \leq r_{max}, \\ where \;\; x_i \neq x_j, i, j = 1, 2...N, \quad v_i, v_l \in \{1, 2.., or P\} \end{array}\right\} \quad (6)$$

Where r_{max} is the maximum risk/discomfort factor that should be maintained by the salesman in the entire tour to avoid unwanted situation.

Proposed 4 Dimensional TSP(4DTSP)

(a) Four Dimensional TSP(4DTSP)

Let $c(i, j, r, k)$ and $t(i, j, r, k)$ be the cost and time respectively for travelling from i-th city to j-th city by the r-th route using k-th type conveyance. Then the salesman has to determine a complete tour (x_1, x_2, ...,x_N, x_1) and corresponding available route types (r_1, r_2, ..., r_s) with conveyance types (v_1, v_2, ..., v_p) to be used for the tour, where $x_i \in \{1, 2, ..N\}$ for $i = 1, 2, ..., N$, $r_i \in \{1, 2, ..s\}$ and $v_i \in \{1, 2, ..p\}$ for $i = 1, 2, ..., N$ and all x_i's are distinct. Then the problem can be mathematically formulated as:

$$\left.\begin{array}{c} \text{minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, r_i, v_i) + c(x_N, x_1, r_l, v_l), \\ \text{subject to} \quad \sum_{i=1}^{N-1} t(x_i, x_{i+1}, r_i, v_i) + t(x_N, x_1, r_l, v_l) \leq t_{max}, \\ where \;\; x_i \neq x_j, i, j = 1, 2...N, \quad r_i, r_l \in \{1, 2.., or\ s\}, \\ v_i, v_l \in \{1, 2.., or\ p\} \end{array}\right\} \quad (7)$$

(b) 4DTSP with Restricted Path and Time constraint

In real life, it is seen that in all stations, all types routes may not be available due to the geographical position of the station,weather conditions, etc. So it is more realistic, that restricted routes be considered to travel different stations. Let $c(i, j, r, k)$ and $t(i, j, r, k)$ be the cost and time respectively for travelling from i-th city to j-th city by the r-th route using k-th type conveyance. Then the salesman has to determine a complete tour (x_1, x_2, ...,x_N, x_1) and corresponding available route types (r_{m1}, r_{m2}, ..., r_{ms}) with conveyance types (v_{q1}, v_{q2}, ..., v_{qp}) providing maximum available $s_1(\leq s)$ and $p_1(\leq p)$ types of routes and conveyances to be used for the tour, where $x_i \in \{1, 2, ..N\}$ for $i = 1, 2, ..., N$, $r_{mi} \in \{1, 2, ..s_1\}$ and $v_{qi} \in \{1, 2, ..p_1\}$ for $i = 1, 2, ..., N$ and all x_i's are distinct. Then the problem can be mathematically formulated as:

$$\left.\begin{array}{c} \text{minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, r_{mi}, v_{qi}) + c(x_N, x_1, r_{ml}, v_{ql}), \\ \text{subject to} \quad \sum_{i=1}^{N-1} t(x_i, x_{i+1}, r_{mi}, v_{qi}) + t(x_N, x_1, r_{ml}, v_{ql}) \leq t_{max}, \\ where \;\; x_i \neq x_j, i, j = 1, 2...N, m = 1, 2, ...s_1 \, , q = 1, 2, .., p_1, \\ r_{mi}, r_{ml} \in \{1, 2.., or \;\; s_1\}, v_{qi}, v_{ql} \in \{1, 2.., or \;\; p_1\}, \end{array}\right\} \tag{8}$$

Historical Review of Uncertain TSPs

The traditional TSP studies mentioned above are all assumed in deterministic environment. However, in the real world, TSP situations are often in deterministic, some or all of the TSPs parameters are not known with certainty at the moment we have to make decision. With the great improvement of probability theory, the stochastic model has been widely used in many relevant TSPs to represent the indeterminacy, including the consideration of probability in the presence of customers Jaillet et al. [17], the demand level Bertsimas et al. [3], the travel time Kao et al. [19], and the service time at customers site Chang et al. [4], usually assuming a known distribution governs some of the problems parameters. Sepideh Fereidouni [36] used a fuzzy multi-objective linear programming. Chaudhuri et al. [6], used a Fuzzy multi-objective linear programming for TSP.

Review of Different Heuristic Methods for TSPs

D.B. Fogel implemented one of the first successful evolutionary optimisation approaches to the TSP which he described in an evolutionary approach to the travelling salesman problem [13] in 1988. In this paper he outlined an alternative to the genetic operators which Holland [30] proposed in 1987. Xing et al. [38] presented a hybrid approach which combines an improved GA and optimization strategies for solving the asymmetric TSP (ATSP). Bai et al. [2] proposed a max-min ant colony optimization method for the solution of ATSPs bridging the gap between hybridization and theoretical analysis. Jula et al. [18] considered a routing problem with stochastic travel times and time windows estimating means and variances of arrival times at nodes and removing routes that are dominated by others. Wang et al. [37] proposed an approximate method on sparse graph for TSP, Nagata et al. [29] developed a new GA for asymmetric TSP, Che et al. [7] considered genetic simulated annealing ant colony systems with PSO to solve TSP, Albanyrak et al. [1] developed a new mutation operator to solve TSP by GA, Xu et al. [39] solved multi-objective problem with power station operation, Elaoud et al. [10] proposed multiple crossover and mutation operators with dynamic selection scheme in MOGA for multi- objective TSP (MOTSP), Lust et al. [24, 25] presented two-phase Pareto local search (2PPLS) for bi objective TSP, Filippi et al. [11] considered a Pareto ϵ approximation named as ABE algorithm for MOTSP, Samanlioglu et al. [34] proposed weakly Pareto optimal solutions for symmetric MOTSP with memetic random-key GA, Zhou et al. [42] considered multi-objective estimation of distribution algorithm based on decomposition (MEDA/D) for some particular MOTSPs. Paquete et al. [31] analyze algorithmic components of stochastic local search algorithms for the multiobjective travelling salesman problem.

Motivation and Objectives of the Thesis
Motivation:

Soft Computing (SC) is a widely used technique in present research of the optimization. Now a days SC is used to design the complex real world problems. Again it is a part of artificial intelligence. Evolutionary computing techniques are a part of SC. GA, ACO and PSO are the most popular evolutionary approaches for designing and solving the complex optimization problems in present phenomena. Genetic algorithms are robust adaptive optimization techniques based on a biological paradigm. They perform efficient search on poorly-defined spaces by maintaining an ordered pool of strings that represent regions in the search space. Again it attempts to increase the effectiveness of the search techniques. GAs have already been applied to several difficult search problems. Similarly, ACO and PSO are the biologically inspired SC techniques used to solve the complex decision making problems. The hybridization of these methods is much effective for solving the problems. All these SC techniques elaborately are used mainly on continuous optimization but few methods are applied in discrete optimization problems also. So there are limited research works in discrete cases. Particularly for GA, a lot of well known operators are available to solve both continuous and discrete optimization problems. There is lot of scope of developing the different GA operators and also the hybridization of GA, ACO and PSO for the optimum solutions of NP-hard problems. **This prompted us to take up research works to bring the different variations in the GA, ACO and PSO operators and to make different combinations of GA, ACO and PSO to derive the near optimum solution of discrete NP-hard problems.**

Normally two-dimensional TSPs are available in the literature. But, in real-life, three and four-dimensional (-3D and -4D) TSPs are in vogue. In 3DTSP, different conveyance available at different nodes are used by the salesman for minimum cost. In 4DTSP, in addition to availability of conveyances of the nodes, there are different paths for travel between the nodes. Though few researchers have considered 3DTSPs (Changder et al.,[5]) but till now, none formulated 4DTSPs. These TSPs have the wide application for medical representative, network routing, transport, logistical problems and electronic manufacturing field, etc. Again, these NP-hard problems can be formulated and solved in different imprecise (fuzzy, rough, etc.) environments. In these cases, the costs, distances, time, etc. of the system may be fuzzy, rough,, fuzzy-rough, etc.

So the above mentioned gaps and considerations motivated us to design different types of GA operators and to develop different hybridization of GA, ACo and PSO for the solution of the above mentioned TSPs. During the research period, it is observed that to solve the discrete optimization problems by SC techniques particularly GA, ACO and PSO, there is a lot of scope to design new operators with different uncertain parameters and new hybridization technique.

The available data of the travelling systems, such as costs, time, risk/discomfort and safety factors etc. are not always exact or precise but are uncertain or imprecise due to uncertainty in judgment, insufficient information, conditions of road, weather condition, etc. and uncertainty of availability of travelling vehicles also. **This motivated us to consider some innovative TSPs in uncertain environments like fuzzy, random, interval valued, rough, bi-fuzzy, bi-rough, bi-random, random-fuzzy, random-rough, fuzzy-random and fuzzy-rough etc.**

Objective of the Thesis:

The main objectives of the presented thesis are:

- **To formulate different types of operators of GA:**
  Some innovative and useful selection operators of GA such as probabilistic, fuzzy

age based, extended fuzzy age based, rough age based, rough extended age based and rough set based pheromone classification for ACO are developed. Again new crossover as Comparison crossover and adaptive crossover are modeled for GA. Many virgin mutation operators such as node oriented, p_m (probability of mutation) dependent and generation dependent mutations are developed to deal with solid TSPs.

- **To formulate hybridization of ACO-GA and ACO-PSO-GA:**
  To day in several cases, hybrid methods are effective in designing intelligent systems. In real world applications, such a fusion between different evolutionary approaches have always a concrete response to improve performance, to reduce computational burden, or to lower the total product/process cost. Here some needful combinations of rough set based pheromone updated ACO with GA and three well known evolutionary approaches ACO, PSO and GA are merged with modified version.

- **To formulate different types multi-objective GA:**
  Though several research works have been done about multi-objective GA, however there are some scopes of research in this field in particular to solve solid TSPs. As in every real world problem contains some uncertainty, the present investigation includes new improved of impreciseness of the multi-objective GA. Here we tried to introduce two different types of uncertainty i.e. fuzzy and rough in multi-objective GA, i.e. imprecise MOGA (iMOGA) and these are used to Rough MOGA (RMOGA) and solve solid TSPs with cost and time as two objectives along with a constraint.

- **To formulate different types of TSPs:**
  Here we have formulated some different types of TSPs models such as solid TSPs i.e. three dimensional TSPs (3DTSPs) where a traveler can choose a conveyance from different types of available vehicles to journey from one city to another city. Also we considered some constraints as time, cost, risk and safety, etc. in constraint solid TSP (CSTSP). Again a new model is designed such as constrained solid TSP with restricted conveyance (CSTSPwR). For the first time, four dimensional TSPs (4DTSPs) are modeled considering different paths from one city to another city, here several also vehicles are also available along each route.

- **To consider different types of uncertainties in TSPs:**
  Decision making with uncertainty is an emerging area. Though few research works have been done on TSPs in fuzzy and random environments, however there are lots of scopes to do research in this area. In the present investigation, several uncertainties are considered for 3DTSPs and 4DTSPs such as fuzzy with possibility, necessity, expected value method and credibility approach, rough with expectation and trust measure, random with chance constraint programming approach, interval valued with different objective model, bi-fuzzy, bi-rough, bi-random, random-fuzzy, random-rough, fuzzy-random, fuzzy-rough environments with their different approaches. Again for the first time, trust measures are extended with five-point scale and seven point scale.

Organization of the Thesis:

The proposed thesis has been divided into following four parts and eight Chapters.

Part-I: Introduction and Methods/ Techniques

In the first chapter, contain a brief introduction of the thesis. The general structure of the development of soft computing techniques, combinatorial optimization, TSP as NP hard problem, different uncertain environments and history of SC techniques to solve TSP in different hybrid uncertain environments have been discussed. In the second chapter, a brief over view about the heuristic computing are presented. In chapter-3, here some mathematical prerequisite of the uncertainty is presented.

Chapter-1

Introduction and Methods/Techniques

This chapter contains a brief introduction giving an overview of the development on soft computing methods with combinatorial optimization in different hybrid uncertain environments.

Chapter-2

Some Specific Heuristics

In this chapter, study the heuristics such as Genetic Algorithm (simple GA), Fast and Elitist Multi-Objective Genetic Algorithm, Non-dominated Sorting Genetic Algorithm, Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO) briefly with their merit and demerits. Here identified the lac of these algorithms to solve for particular discrete optimization problems. Also hybridization of two or more swarm heuristics with their literature review are given.

Chapter-3

Some Uncertain Environments

In this chapter, briefly discussed different uncertainty with their combinations as interval valued, random, fuzzy, rough, Bi-random, Bi-rough, Bi-fuzzy, Fuzzy-rough, rough-fuzzy, fuzzy-random, random-fuzzy, rough-random and random-rough variables. Here few proposed mathematical extension of the uncertainty variables are presented.

Part-II: Single Objective Optimization Using Single/Multi Heuristic Methods

Chapter-4

Single Objective Optimization using Single Heuristic Methods

In this chapter, presents the five model about the proposed Genetic algorithm different operators such as selection, crossover and mutation with introducing solid TSPs under crisp, fuzzy, random, random-fuzzy, fuzzy-random, bi-random, bi-rough environment are developed and solved by the proposed models.

Model-4.1: An Improved Genetic Algorithm and Its Application in Constrained Solid TSP in Uncertain Environments

In this investigation, a GA is to proposed to solved the solid TSPs under different uncertain environments. Here proposed an improved genetic algorithm (IGA) to solve Constrained Solid Travelling Salesman Problems (CSTSPs) in crisp, fuzzy, rough, and fuzzy-rough environments. The algorithm is model with a combination of probabilistic selection, cyclic crossover, and nodes-oriented random mutation. Here, CSTSPs in different uncertain environments have been designed and solved by the proposed algorithm. A CSTSP is usually a travelling salesman problem (TSP) where the salesman visits all cities using any one of the conveyances available at each city under a constraint say, safety constraint. Here a number of conveyances are used for travel from one city to another. . The salesman desires to maintain certain safety level always to travel from one city to another and a total safety for his entire tour. Costs and safety level factors for travelling between the cities are different. The requirement of minimum safety level is expressed in the form of a constraint. The safety factors are expressed by crisp, fuzzy, rough, and fuzzy-rough numbers. The problems are formulated as minimization problems of total cost subject to crisp, fuzzy, rough, or fuzzy-rough constraints. This problem is numerically illustrated with appropriate data values. Optimum results for the different problems are presented via IGA. Moreover, the problems from the TSPLIB (standard data set) are tested with the proposed algorithm with some statistical test.

Model-4.2: Constrained Solid Travelling Salesman Problem Using Adaptive Genetic Algorithm in Uncertain Environment

In this model, an Adaptive Genetic Algorithm (AGA) is developed to solve constrained solid travelling salesman problems (CSTSPs) in crisp, fuzzy and rough environments. In the proposed AGA, we model it with probabilistic selection and proposed a virgin adaptive crossover with random mutation. Present model, CSTSPs are illustrated numerically by some empirical data using this algorithm. In each environment, some sensitivity studies due to different risk/discomfort factors and other system parameters are presented.

Model-4.3: A Modified Genetic Algorithm for solving uncertain Constrained Solid Travelling Salesman Problems

The present investigation, design a Modified Genetic Algorithm (MGA) is developed to solve Constrained Solid Travelling Salesman Problems (CSTSPs) in crisp, fuzzy, random, random-fuzzy, fuzzy-random and bi-random environments. In the developed MGA, a probabilistic selection technique and a comparison crossover are used along with conventional random mutation. In CSTSP, along each route, there may be some risk/discomfort in reaching the destination and the salesman desires to have the total risk/discomfort for the entire tour less than a desired value. Here we model the CSTSP with traveling costs and route risk/discomfort factors as crisp, fuzzy, random, random-fuzzy, fuzzy-random and bi-random in nature. A number of benchmark problems from standard data set, TSPLIB are tested against the existing Genetic Algorithm (with Roulette Wheel Selection (RWS), cyclic crossover and random mutation) and the proposed algorithm and hence the efficiency of the new algorithm is established. In this model, CSTSPs are illustrated numerically by some empirical data using this algorithm. In each environment, some sensitivity studies due to different risk/discomfort factors and other system parameters are presented.

Model-4.4: A Rough Genetic Algorithm for Constrained Solid TSP with Interval Valued Costs and Times

This model presents a Rough Set based Genetic Algorithms (RSGAs) to solve constrained Solid Travelling Salesman Problems (CSTSPs) with restricted conveyances (CSTSPwR) having uncertain costs and times as interval values. In the proposed RSGAs, a rough set based age dependent selection technique and an age oriented min-point crossover are used along with three types of p_m- dependent random mutations. A number of benchmark problems from standard data set, TSPLIB are tested against the proposed algorithms and existing standard GA (SGA) and hence the efficiency of the new algorithms are established. Here CSTSP is a STSP with a constraint (say time constraint). We have modelled CSTSPwRs where some conveyances are not allowed to run in some particular routes. CSTSPwRs are formulated as constrained linear programming problems and solved by both proposed RSGAs and SGA. These are illustrated numerically by some empirical data and the results from the above methods are compared. Statistical significance of the proposed algorithms are demonstrated through statistical analysis using standard deviation (SD). Moreover, the non-parametric test, Friedman test is performed with the proposed algorithms. In addition, a Post Hoc paired comparison is applied and the out performance of the RSGAs are established.

Model-4.5: A Rough extended Genetic Algorithm for Solving Constrained Solid Travelling Salesman Problem Under Bi-Fuzzy Coefficients

In this model, a Rough extended Genetic Algorithm (ReGA) is proposed to solve constrained solid travelling salesman problems (CSTSPs) in crisp and bi-fuzzy coefficients. In the proposed ReGA, developed a rough set based selection (7-point scale) technique and comparison crossover with improved generation dependent mutation. The costs and risk/discomforts factors are in the form of crisp, bi-fuzzy in nature. Here CSTSPs are illustrated numerically by some standard test data from TSPLIB using ReGA. In each environment, some statistical significance studies due to different risk/discomfort factors and other system parameters are presented with some statistical test.

Chapter-5

Single objective Optimization Using hybrid heuristic Techniques

In this chapter two hybrid heuristics are developed solved proposed four dimensional TSPs under bi-fuzzy and bi-rough coefficients. The first model is the combinations of proposed ACO and GA with rough set based pheromone classifier. For the second model hybridize with another swarm intelligent approach PSO and formed a ACO-PSO-GA based model.

Model-5.1: An Intelligent Hybrid Algorithm for 4- Dimensional TSP

Present model described, a hybridized algorithmic approach to solve 4- dimensional Travelling Salesman Problem (4DTSP) where different paths with various number of conveyances are available to travel between two cities. The algorithm is a hybridization of rough set based ant colony optimization (rACO) with proposed genetic algorithm (GA). The initial solutions are produced by ACO which act as a selection operation of GA after it a GA is developed with a virgin extended rough set based selection (7-point scale), comparison crossover and generation dependent mutation. The said hybrid algorithm rough set based Ant Colony Optimization (rACO) with Genetic Algorithm (rACO-GA) is tested against some test functions and efficiency of the proposed algorithm is established. The 4DTSPs are formulated with crisp and bi-fuzzy costs. In each environment, some statistical significant studies due to different time constraint values and other system parameters are presented. The models are illustrated with some numerical data.

Model-5.2: A new Evolutionary Hybrid Algorithm for restricted 4-Dimensional TSP (r-4DTSP) in Uncertain Environment

In this model, we proposed an hybridized three known soft computing technique to solve a restricted 4- dimensional TSP (r-4DTSP). Here some restrictions on paths and conveyances are imposed. The developed hybrid methods combines the ant colony optimization (ACO) and swap operator based particle swarm optimization (PSO) with modified genetic algorithm (GA). The initial solutions are produced by ACO which used as swarm in PSO then a modified GA with virgin selection, comparison crossover and generation dependent mutation. The said hybrid algorithm (ACO-PSO- GA) is tested against some test functions and efficiency of the proposed algorithm is established. The r-4DTSPs are considered with crisp and bi-rough costs. In each environment, some statistical significant studies due to different time constraint values and other system parameters are presented. The models are illustrated with some numerical data.

Part-III: Multi Objective Optimization Using a Heuristic Methods

Chapter-6

Multi- objective optimization using heuristic algorithm

In this chapter contain two multi-objective GA with rough and fuzzy selection operators is developed and solve solid TSP with cost and time as objectives under different hybrid uncertainty.

Model-6.1: An imprecise Multi-Objective Genetic Algorithm for uncertain Constrained Multi-Objective Solid Travelling Salesman Problem

In this model, an imprecise Multi-Objective Genetic Algorithm (iMOGA) is developed to solve Constrained Multi-Objective Solid Travelling Salesman Problems (CMOSTSPs) in crisp, random, random-fuzzy, fuzzy-random and bi-random environments. In the proposed iMOGA, 3 - and 5 - level linguistic based fuzzy age oriented selection, probabilistic selection and an adaptive crossover are used along with a new generation dependent mutation. In each environment, some sensitivity studies due to different risk/discomfort factors and other system parameters are presented. To test the efficiency, combining same size single objective problems from standard TSPLIB, the results of such multi-objective problems are obtained by the proposed algorithm, simple MOGA (Roulette wheel selection, cyclic crossover and random mutation), NSGA-II, MOEA-D/ACO and compared. Moreover, a statistical analysis (Analysis of Variance) is carried out to show the supremacy of the proposed algorithm.

Model-6.2: A Rough Multi-Objective Genetic Algorithm for uncertain Constrained Multi-Objective Solid Travelling Salesman Problem

The present model proposed a Rough Multi-Objective Genetic Algorithm (R-MOGA) to solve Constrained Multi-Objective Solid Travelling Salesman Problems (CMOSTSPs) in rough, fuzzy rough and random rough environments. In the proposed R-MOGA, '3 - and 5 - level linguistic based rough age oriented selection', 'adaptive crossover' are used along with a improved generation dependent mutation. In CMOSTSP, along each route, there may be some risk/discomfort in reaching the destination and the salesman desires to have a total risk/discomfort for the entire tour less than a desired value. Here we model the CMOSTSP with travelling costs and times as two objectives and a constraint for route risk/discomfort factors. The costs, times and risk/discomfort are rough, fuzzy rough and random rough in nature. CMOSTSPs are illustrated numerically by some empirical data using this algorithm. To test the efficiency, combining same size single objective problems from standard TSPLIB, the results of the such multi-objective problems are obtained by the proposed algorithm, simple MOGA and NSGA-II compared. A statistical analysis (Analysis of Variance) is carried out to show the efficiency of the proposed algorithm.

Part-IV: Summary and Future Research Scope

Chapter-7

Summary and Future Research scope

In this chapter a short summary with a brief future research are discussed.

Summary and Future Research Scope

In this dissertation, main objectives are (i) to develop/modify some evolutionary methods, specially Genetic Algorithm (GA), Ant Colony Optimization (ACO) and Particle Swarm Optimization (PSO), (ii) to develop some hybrid evolutionary methods connecting GA, ACO and PSO and (iii) to formulate some new uncertain (random and imprecise) singe/ multi-objective TSP problems and to solve them using the developed evolutionary methods (GAs) and hybrid evolutionary methods.

Here, for the first time, constrained single/multi-objective some 3D- and 4D- TSPs have been formulated in crisp, fuzzy, random, bi-random, bi-fuzzy, rough, bi-rough, fuzzy-rough, fuzzy-random, random-fuzzy, random-rough, etc, environments. Some uncertain risk/safety constraints along the routes and time constraints for the tour are also imposed. These virgin problems have been solved by developed evolutionary and hybrid evolutionary methods. Restrictions on vehicles and paths are imposed in 3D- and 4D- TSPs respectively. In this thesis, in developing different GAs, nine (9) different types of selections such as probabilistic selection, probability of selection parameter, fuzzy age based, fuzzy -extended age based, rough age based (3, 5 and 7 point scale), rough pheromone based, three (3) types of crossover such as adaptive crossover, comparison crossover, min-point crossover and five (5) types of mutations such as nodes oriented, generation dependent (two types), fixed point location and random mutations are used. Also two (2) types of hybrid evolutionary algorithms such as ACO-GA and ACO-PSO-GA have been developed and used. Here, in ACO, rough pheromone has been defined and used. In PSO, swap sequence based updatation of velocities and positions has been used. To the best of my knowledge, none developed and used these operators before.

In total, nine (9) virgin constrained TSP models have been formulated and solved.

These new algorithms have been tested against standard problems from TSPLIB to establish the efficiency of the developed techniques. The results from these methods are statistically tested. The statistical tests include Mean, SD, error analysis, ANOVA, Firedman Test and Post hoc paired comparison.

The thesis has been divided into four parts. Part-I contains three chapters- chapter-1 contains Introduction, chapter-2 preliminaries of algorithms and chapter-3 different uncertainties. Part-II contains two chapters where chapter-4 and chapter-5 contains single objective constraints respectively 3D-TSP and 4D-TSP problems solved by single (GA) and hybrid evolutionary (ACO-GA and ACO-PSO-GA) techniques. In chapter-6, multi-objective TSP problems are formulated and solved by developed evolutionary algorithm. All these TSP models are formulated in uncertain environments.

In chapter-7, Part-IV summary and future extensions along with bibliography and index are given.

Limitations and Future Extension

The present investigation has been confined to the development of different types of GA, ACO, PSO and two hybridization of GA, ACO and PSO in uncertain environments. There are other several evolutionary algorithms such as Artificial Bees Colony (ABC), Tabu search, etc., which can also be extended in uncertain (random and imprecise) environments and can be used tio solve the Np-hard TSP, CSP, VRP, etc., problems.

In three dimensional TSPs with conveyances, we have assigned a conveyance arbitrarily during each crossover and mutation for the optimum selection of the routes. This is a limitation of the present CSTSPs. The formulated CSTSPs can be extended to include some more features /restrictions such as 4DTSP to include/omit some specific routes, time windows, multi-TS, profit maximization, etc. In the literature, these several types of TSPs such as multiple TSP, MAx TSP etc., which can be developed in different uncertain environments not dealt in this thesis and solved by the proposed algorithms.

Part-V: Bibliography and Index

Chapter-8

Bibliography and Index

In this chapter Bibliography and Index are presented.

Bibliography

[1] Albanyrak, M., Allahverdi, N., *Development of a new mutation operator to solve the TSP by genetic Algorithm*, Expert Systems with Application, 38(3)(2011), 1313-1320.

[2] Bai, J., Yang, G-K. Chen, Y-W. Hu, L-S, Pan, C-C., *A model induced max-min ant colony optimization for asymmetric travelling salesman problem*, Applied Soft Computing, 13(2013), 1365-1375.

[3] Bertsimas, D. J., Simchi-Levi, D., *A new generation of vehicle routing research:Robust algorithms, addressing uncertainty*. Operations Research, 44(2)(1996), 286-304.

[4] Chang, T.-S., Wan, Y.-W., Ooi, W. T., *A stochastic dynamic traveling salesman problem with hard time windows*, European Journal of Operational Research, 198(3)(2009), 748-759.

[5] Changdar, C., Maiti, M. K., Maiti, M., *A Constrained solid TSP in fuzzy environment:two heuristic approaches, Iranian Journal of Fuzzy System*, 10(1)(2013), 1-28.

[6] Chaudhuri1, A., De, K., *A Study of Traveling Salesman Problem Using Fuzzy Self Organizing Map*, Traveling Salesman Problem, Theory and Applications, Prof. Donald Davendra (Ed.), ISBN: 978-953- 307-426-9, InTech.

[7] Che, S. M., Ohiem, C. Y., *Solving TSP based on the genetic simulated annealing ant colony system with PSO*, Expert Systems with Application, 38 (12)(2012), 14439-14450.

[8] Dantzig, G. B., *Linear Programming and Extensions*, Princeton University Press, New Jersey (1963).

[9] Dorigo, M., *Optimization, Learning and Natural Algorithms*, Ph D thesis, Politecnico di Milano, 1992.

[10] Elaoud, S., Teghem, J., Loukil, T., *Multiple crossover genetic algorithm for the multi- objective travelling salesman problem*, Electronics notes in Discrete Mathematics, 36(2010), 939-946.

Automation, pp. 1186-1192, 1992.

[11] Filippi, C., Stevanato, E., *Approximation schemes for bi-objective combinatorial optimization and their application to the TSP with profits*, Computers & Operations Research, 40(2013), 2418-2428.

[12] Focacci, F., Lodi, A., Milano, M., *A hybrid exact algorithm for the TSPTW*, Inform Journal on Computing, 14(4)(2002), 403-417.

[13] Fogel, D. B., *An Evolutionary Approach to the Traveling Salesman Problem*, Biological Cybernetics, vol. 60, pp. 139-144, 1988

[14] Goldberg, D., *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison Wesley, MA, USA, (1989).

[15] Holland, H. J., *Adaptation in Natural and Artificial Systems*, University of Michigan, (1975).

[16] Jang, J. S., Sun, C., *Neuro-fuzzy modeling and control*, Proceedings of IEEE 83(3)1995, 378-406.

[17] Jaillet, P., *A priori solution of a traveling salesman problemin which a random subset of the customers are visited*. Operations Research, 36(6)(1988),929-936.

[18] Jula, H., Dessouky, M., Ioannou, P. A., *Truck route planning in non-stationary stochastic networks with time windows at customer location*, IEEE Transactions on Intelligent Transportation Systems, 7(1)(2006), 51-52.

[19] Kao, E., *A preference order dynamic program for a stochastic travelling salesman problem*, Operations Research, 26(1978), 1033-1045.

[20] Karmakar, S., Mahato, S. K., Bhunia, A.K., *Interval oriented multi-section techniques for global optimization*, Journal of Computational and Applied Mathematics, 224(2)(2009), 476-491.

[21] Kennedy, J., Eberhart, R., *Particle swarm optimization*, Proceedings of the IEEE International Conference on Neural Networks, Perth, Australia, 1 (1995) 1942-1945.

[22] Koza, J. R., *Genetic Programming: On the Programming of Computers by Means of Natural Selection*, Cambridge, MA, USA: MIT Press, 1992.

[23] Liaw, C. F., A hybrid genetic algorithm for the open shop scheduling problem, European Journal of Operational Research 124(1)(2000), pp. 2842

[24] Lust, T., Jaszkiewicz, A., *Speedup techniques for solving large-scale bi objective TSP*, Computers & Operations Research, 37(2010), 521-533.

[25] Lust, T., Teghan, J., *Two phase Pareto local search for bi-objective travelling salesman problem*, Journal of Heuristics, 16(3)(2010), 475-510.

[26] Majumder, S., Bhunia, A. K., *Genetic algorithm for asymmetric traveling salesman problem with imprecise travel times*, Journal of Computational and Applied Mathematics, 235(9)(2011), 3063-3078.

[27] Mestria, M., Ochi, L. S., Martins, S. L., *GRASP with path relinking for the symmetric Euclidean clustered traveling salesman problem*, Computers & Operations Research, 40(12)(2013), 3218-3229.

[28] Moon, C. K., Choi, J., Seo, Y. G., *An efficient genetic algorithm for the traveling salesman problem with precedence constraints*, European Journal of Operational Research, 140(2002), 606-617.

[29] Nagata, V., Soler, D., *A new GA for asymmetric TSP*, Expert Systems with Application, 39(10)(2012), 8947-8953.

[30] Oliver, I. M., Smith, D. J., Holland, J. R., *A study of permutation crossover operators on the traveling salesman problem*, Proc. 2nd International Conference on Genetic Algorithms and their Applications, pp. 224-230, 1987.

[31] Paquete, L., Stutzle, T., *Design and analysis of stochastic local search for the multi-objective traveling salesman problem*, Computers & Operations Research, 36(9)(2009), 2619-2631.

[32] Petersen, H. L., Madsen, O. B. G., *The double travelling salesman problem with multiple stack - Formulation and heuristic solution approaches*, European Journal of Operational Research, 198(2009), 339-347.

[33] Rakke, J. G., Christiansen, M., Fagerholt, K. Laportei, G., *The Traveling Salesman Problem with Draft Limits*, Computers and Operations Research, 39(9)(2012), 2161-2167.

[34] Samanlioglu, F., Ferrel Jr, W. G., Kurz, M. E., *A memetic random-key genetic algorithm for a symmetric multi-objective travelling salesman problem*, Computer & Industrial Engineering, 55(2008), 439-449.

[35] Sanchez, E., Squillero, G., Tonda, A., *Industrial Applications of Evolutionary Algorithms*, vol. 34 of Intelligent Systems Reference Library, Springer, 2012.

[36] Sepideh, F., *Solving traveling salesman problem by using a fuzzy multi objective linear programming*, African Journal of Mathematics and Computer Science Research, 4(11)(2011), 339-349.

[37] Wang, Y., *An Approximate method to compute a sparse graph for travelling salesman problem*, Expert Systems with Application, (2015) In press.

[38] Xing, L. N., Chen, Y-W., Yang, K-W., Hou, F. Shen, X-S., Cai, H-P., *A hybrid approach combining an improved genetic algorithm and optimization strategies for the asymmetric travelling salesman problem*, Artificial Intelligence, 21 (2008), 1370-1380.

[39] Xu, J., Tao, Z., *A class of multi-objective equilibrium chance maximization model with twofold random phenomenon and its application to hydropower station operation*, Mathematics and Computers in Simulation, 83(1)(2012), 11-33.

[40] Yuan, S., Skinner, B., Huang, S., Liu, D., *A new crossover approaches for solving the multiple travelling salesman problem using genetic algorithms*, European Journal of Operational Research, 228(2013), 72-82.

[41] Zadeh, L. A., *The concept of a linguistic variable and its application to Appromximate Resoning -I*, Information Sciences, 8(1975), 199-249.

[42] Zhou, A., Gao, F., Zhang, G., *A decomposition based estimation of distribution algorithm for multi-objective travelling salesman problem*, Computers and Mathematics with Applications, 66(2011), 1857-1868.