2017

M.Sc. Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X (OR)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper

[Advanced Optimization and Operations Research–II]

Answer Q. No. *11* and any *six* from the rest.

[Calculator may be used]

1. (a) Obtain the probability p_n for n customers in the system for the (M/M/1) : (∞/FCFS/∞) queuing model. 4

(b) What do you mean by finite and infinite queue ?

What information can be obtained by analyzing a queuing system ? Explain any two parameters used in queuing model. 10+6

2. (a) Explain the following terms related to information theory :

transmitter, receiver, communication channel and noise.

Draw a diagram mention the relationship among them along with decoder and encoder. 4

(b) What do you mean by memory lens channel and channel matix ? 2

(c) Show that the entrophy of the following probability distribution is $2-\left(\frac{1}{2}\right)^{n-2}$:

Event :	x_1	x_2	$x_3 \cdots x_i \cdots x_{n-1}$	x_n
Probability :	$\frac{1}{2}$	$\frac{1}{2^2}$	$\frac{1}{2^3} \cdots \frac{1}{2^i} \cdots \frac{1}{2^{n-1}}$	$\frac{1}{2^n}$

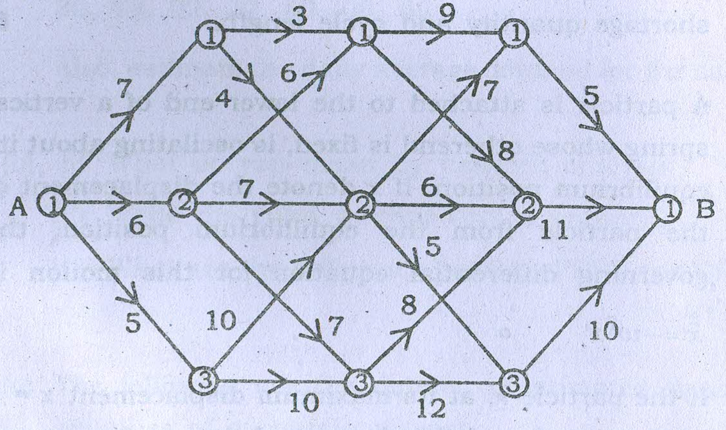
(d) A transmitter has a character consisting of five letters (x_1, x_2, ..., x_5) and the receiver has a character consisting of four letters (y_1, y_2, y_3, y_4). The joint probability for the communication is given below :

$p(x_i, y_j)$	y_1	y_2	y_3	y_4
x_1	0.25	0	0	0
x_2	0.10	0.30	0	0
x_3	0	0.05	0.10	0
x_4	0	0	0.05	0.10
x_5	0	0	0.05	0

Determine the entropies $H(X)$, $H(Y)$ and $H(X, Y)$. 6

3. (a) Find the shortest path from the vertex A to the vertex B along the edges joining various vertices lying between A and B shown below :

The numbers associated with the edges represent edge weights. Dynamic programming method may be used. 8



(b) Use dynamic programming method to prove that $\sum_{i=1}^{n} p_i \log p_i$, subject to $\sum_{i=1}^{n} p_i = 1$, is minimum when $p_1 = p_2 = \cdots = p_n = \frac{1}{n}$. 10+6

4. (a) In an inventory model, demand is uniform, production rate is infinite, shortages are allowed and fully backlogged, lead time is zero, the planning horizon is infinite.

For this system, find the optimum order quantity and minimum cost. 10

(b) The demand form item is 18000 units per year. The inventory carrying cost is Rs. 1.50 per unit time and the cost of shrotage is Rs. 5.00. The ordering cost is Rs. 500. Assuming that the replacenishment rate is instantaneous, determine the optimum order quantity, shortage quantity and cycle length. 6

5. (a) A particle is attached to the lower end of a vertical spring whose otherend is fixed, is oscilating about its equilibrium position. If x denote the displacement of the particle from the equillibrium position, the governing differential equation for this motion is

$$\ddot{x} = -w^2 x$$

If the particle is at its maximum displacement $x = a$ at time $t = 0$ and this instant of time, a force u per

unit mass is applied to the particle in order to bring the particle to rest when its displacement is zero, find such a force u. 8

(b) Describe a method to generate random numbers. Generate 6 random numbers by using your proposed method. Also, write merits and demerits of the proposed method. 8

6. (a) A bakery keeps stock of a popular brand of cake. Previous experience shows the daily demand pattern for the item with associated probabilities, as given below :

Daily demand :	0	10	30	40	50
Probability :	0.01	0.20	0.50	0.12	0.02

Use the following sequence of random numbers to simulate the demand fro next 10 days : 20, 25, 30, 80, 79, 92, 32, 40, 09, 52

Also, estimate the daily average demand for the cakes. 8

(b) (i) What do you mean by network analysis

(ii) Write the rules to construct a network.

(iii) What are the difference between PERT and CPM ? 2+3+3

7. (a) The following are the details of estimated times of activities of a contain project.

Activity:	A	B	C	D	E	F
Immediate Predecessor :	–	A	A	B,C	–	E
Estimated Time (Weeks) :	2	3	4	6	2	8

(i) Calculate the earliest start time and earliest finish time for each activity.

(ii) Calculate all three floats for each activity.

(iii) Find the critical path and the project completion time. 8

(b) Find the reliability of a system with two components of which one is a stand-by. The components are connected in parallel. 3

(c) In a system, there are n number of components connected in series with reliability $R_i(t)$ = n, i = 1,2,3,...,n. Find reliability of the system. If $R_1(t) = R_2(t) = \ldots R_n(t) = \ldots = R_n(t) = E^{-\lambda t}$ then find the reliability of the system. The system connected in series consists of three independence parts A, B and C which have MTBF of 100, 400 and 800 hours respectively. Find MtBF of the system and reliability of the system for 30 hours. How much MTBF of the parts A has to be increased to get and improvment of MTBF of the system by 30% ? 5

(Continued)

8. (a) Let M be a continuous pay off function of a continuous game then prove that the following conditions are equivalent

(i) F_0 and G_0 are optimal strategies of players 1 and 2 respectively.

(ii) $E(F,G_0) \le E(F_0,G_0) \le E(F_0,G)$ for all strategies of players 1 and 2 respectively.

(iii) $\int_0^1 M(x',y)dG_0(y) \le E(F_0,G_0) \le \int_0^1 M(x,y')dF_0(x)$ for any $x',y' \in [0,1]$. 8

(b) Minimize $f(x) = \frac{1}{x_1x_2} + 10x_1x_2x_3^{-1} + 20x_2x_3 + x_1x_2$

$x_1, x_2, x_3 \ge 0$

Using geometric programming. 8

9. (a) A company contains 10,000 resistors. When any resistor fails, it is placed. The cost of replacing a resistor individually is Rs. 1.0 only. If all the resistors are replaced at the samne time, the cost per resistor would be reduced to Rs. 0.35. The percentage of surviving resistors say $s(t)$ at the end of month t and the prpbability of failure $p(t)$ during the month t are as follows :

(Turn Over)

t	:	0	1	2	3	4	5	6
$s(t)$	:	100	97	90	70	30	15	0
$p(t)$	:	–	0.03	0.07	0.20	0.40	0.15	0.15

What is the optimal replacement plan ? 8

(b) Find the sequence that minimize that total elapsed time required to complete the following tasks. Each job is processed in the order ABC.

Job	:	1	2	3	4	5	6	7
Machine A	:	12	6	5	11	5	7	6
Machine B	:	7	8	9	4	7	8	3
Machine C	:	3	4	1	5	2	3	4

Find also the idle times of each machines, if any. 8

10. (a) Define a non-cooperative game with n palyers. waht is equilibrium situation ? When two games are said to be strategically equivalent ? Show that strategically equivalent games obey symmetric and transitive properties. 8

(b) What is replacement ? Deduce the optimal replacement policy(s) for items whose running cost increaes with time in discrete units and valuye of money remain constant during a period. 8

11. Answer any one questions :

(a) Show that the entropy function is continuous. 4

(b) Define psynomial and degree of difficulty in connection with geometric programming. 4

2017

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X (OM)

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper

Answer any *five* questions. 5×15

1. (a) Deduce the linearized equations of two - dimensional

internal gravity waves propagating in the x - t plane neglecting the coriolis force. 7

(b) Derive the angle between the frontal surface and earth's surface in the atmosphere.

(c) What do you mean by equivalent temperature? What is the concept of CISK? 3

2. (a) Discuss about the phase change of an ideal gas and derive the relation of dependency of latent heat of evaporation with respect to temperature during the phase change of an air parcel. 7

(b) Derive the saturated adiabatic lapse rate of moist air and hence show that it is less than dry adiabatic lapse rate. 5

(c) What do you mean by entropy? Derive the relation between the specific heat constant at constant pressure for moist air and dry air. 3

3. (a) Show that the sum of kinetic energy, potential energy and enthalpy of an air parcel in the atmosphere remains constant when the flow is steady, adiabatic and frictionless. 7

(b) Derive the formula to predict the potential temperature due to advection using finite difference. 5

(c) What is potential temperature? Show that the potential temperature of an air parcel is invariant. 3

4. (a) Derive the Beer's law indicating the relationship between the incident radiative intensity and outgoing transmitted radiative intensity. Hence deduce the coefficient of transmission. 7

(b) Derive the expression for the rate of change of dew-point temperature with height in adiabatically ascending unsaturated moist air. 5

(c) "The pressure is taken as a vertical co-ordinate in the atmosphere" — Justify this statement. 3

5. (a) What is planetary vorticity? Derive the vorticity equation of an air parcel in the atmosphere and interpret each term. 7

(b) Derive the meridional temperature gradient due to Global Circulation and also find meridional temperature and temperature gradient at 45°N latitude at sea level. 5

(c) Derive the geostrophic wind equation in the atmosphere. 3

6. (a) Discuss stability analysis of an air parcel in the atmosphere by Parcel Method. 9

(b) State and prove the Clausius - Clapeyron equation in the atmosphere. 5

(c) Derive Psychrometic equation to measure the actual vapor pressure in terms of dry bulb and wet bulb temperature. 3

7. (a) Derive the barotropic model in the atmosphere. 7

(b) Define enthalpy. Derive the amount of heat to be required to transfer for unit mass of air parcel during the isobaric process. 5

(c) Explain the convergence and divergence in the atmosphere. 3

8. (a) Derive the equation of momentum of an air parcel in the atmosphere in the spherical co-ordinate system. 8

(b) Derive the first law of thermodynamics in the following form $dq = C_p dT - \alpha dp$. 3

(c) Derive the expression of the pressure gradient force in the atmosphere. 4

———