Journal of Physical Sciences, Vol. 22, 2017, 99-106
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 25 December 2017

Some Features of Intuitionistic L- R_0 Spaces

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Received 15 September 2017; accepted 3 November2017

ABSTRACT

R_0 space in intuitionistic L-topological spaces are defined and studied in this paper. We discussed six notions of R_0 space in intuitionistic L-topological spaces and induced certain relationship among them. We also showed that all of these definitions satisfy 'hereditary' property and preserved under one-one, onto and continuous mapping.

Keywords: Intuitionistic L-fuzzy sets, Intuitionistic L-fuzzy point, Intuitionistic L-topology, Intuitionistic L-fuzzy open sets.

1. Introduction

The idea of fuzzy sets and L-fuzzy sets were initially introduced by Zadeh [16] in 1965 and Goguen [12] in 1967 respectively. After then in 1984, intuitionistic fuzzy sets were first published by Attanassov [1] and many works by the same author and his colleagues appeared in the literature [2-4]. Later, this concept was generalized to 'intuitionistic L-fuzzy sets' by Atanassov and Stoeva [5]. Here, we introduced 'intuitionistic L-topology' by using 'intuitionistic L-fuzzy sets' in the sense of Chang [6]. Moreover, we defined possible six notions, investigated some properties and features of R_0 space in intuitionistic L-topological spaces.

2. Notation and preliminaries

Through this paper, X will be a nonempty set, $\emptyset$ be the empty set, and L is a complete distributive lattice with 0 and 1. A , B , … be intuitionistic L-fuzzy sets, t be the intuitionistic topology, τ be the intuitionistic L-topology, $I =$ [0, 1], and the functions $\mu_A: X \to L$ and $\gamma_A: X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of none membership (namely $\gamma_A(x)$).

Now we recall some basic definitions and known results in intuitionistic L-fuzzy sets and intuitionistic L-topological spaces.

Definition 2.1. [16] Let X be a non-empty set and $I = [0,1]$. A fuzzy set in X is a function $u: X \to I$ which assigns to each element $x \in X$, a degree of membership $u(x) \in I$.

Definition 2.2. [13] Let $f: X \to Y$ be a function and u be fuzzy set in X. Then the image $f(u)$ is a fuzzy set in Y which membership function is defined by
$(f(u))(y) = \{sup(u(x)) | f(x) = y\}$ if $f^{-1}(y) \neq \emptyset, x \in X$
$(f(u))(y) = 0$ if $f^{-1}(y) = \emptyset, x \in X$.

Definition 2.3.[12] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An L-fuzzy set in X is a function $\alpha: X \to L$ which assigns to each element $x \in X$ a degree of membership, $\alpha(x) \in L$.

Remark 2.4. Throughout this paper we consider the complete distributive lattice $L = \{0, 0.1, 0.2, \dots, 1\}$ and from the above definitions we show that every L-fuzzy set is also a fuzzy set but converse is not true in general.

Example 2.4.1. Let $X = \{a, b, c\}$ and $L = \{0, 0.1, 0.2, \dots, 1\}$. A function $\alpha: X \to L$ is defined by $\alpha(a) = 0.2, \alpha(b) = 0.5, \alpha(c) = 0$ which is L-fuzzy set and also a fuzzy set.

Example 2.4.2. Let $X = \{a, b, c\}$ and $I = [0, 1]$. A function $u: X \to I$ is defined by $u(a) = 0.25, u(b) = 0.55, u(c) = 0$ which is fuzzy set but not an L-fuzzy set because $0.25, 0.55 \notin L$.

Definition 2.5.[5] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An intuitionistic L-fuzzy set (ILFS for short) A in X is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$. Where the functions $\mu_A: X \to L$ and $\gamma_A: X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of none membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Let $L(X)$ denote the set of all intuitionistic L-fuzzy set in X.Obviously every L-fuzzy set $\mu_A(x)$ in X is an intuitionistic L-fuzzy set of the form $(\mu_A, 1 - \mu_A)$.
Throughout this paper we use the simpler notation $A = (\mu_A, \gamma_A)$ instead of $A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$.

Definition 2.6. [9] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic L-fuzzy sets in X. Then
(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(3) $A^c = (\gamma_A, \mu_A)$
(4) $A \cap B = (\mu_A \cap \mu_B; \gamma_A \cup \gamma_B)$
(5) $A \cup B = (\mu_A \cup \mu_B; \gamma_A \cap \gamma_B)$
(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an ILFS of X and $B = (\mu_B, \gamma_B)$ be an ILFS of Y. Then $f^{-1}(B)$ is an ILFS of X defined by $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ and $f(A)$ is an ILFS of Y defined by $f(A) = (f(\mu_A), 1 - f(1 - \gamma_A))$.

Definition 2.7.[10] An intuitionistic topology (IT for short) on a nonempty set X is a family t of IS's in X satisfies the following axioms:
(i) $\emptyset_{\sim}, X_{\sim} \in t$.
(ii) If $G_1, G_2 \in t$ then $G_1 \cap G_2 \in t$.
(iii) If $G_i \in t$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} G_i \in t$.
Then the pair (X, t) is called an intuitionistic topological space (ITS, for short) and the members of t are called intuitionistic open sets (IOS for short).

Definition 2.8.[11] An ITS (X, t) is called $I - T_0$ space if for all $x, y \in X, x \neq y, \exists$ an IOS $G = (A_1, A_2) \in t$ such that $x \in A_1, y \in A_2$ or $y \in A_1, x \in A_2$.

Definition 2.9.[14] Let $p, q \in L = \{0, 0.1, 0.2, \ldots, 1\}$ and $p + q \leq 1$. An intuitionistic L-fuzzy point (ILFP for short) $x_{(p,q)}$ of X is an ILFS of X defined by

$$x_{(p,q)}(y) = \begin{cases} (p, q) \; if \; y = x, \\ (0,1) if \; y \neq x \end{cases}$$

In this case, x is called the support of $x_{(p,q)}$ and p and q are called the value and none value of $x_{(p,q)}$, respectively. The set of all ILFP of X we denoted it by $S(X)$.
An ILFP $x_{(p,q)}$ is said to belong to an ILFS $A = (\mu_A, \gamma_A)$ of X denoted by $x_{(p,q)} \in A$, if and only if $p \leq \mu_A(x)$ and $q \geq \gamma_A(x)$ but $x_{(p,q)} \notin A$ if and only if $p \geq \mu_A(x)$ and $q \leq \gamma_A(x)$.

Definition 2.10. [14] If A is an ILFS and $x_{(p,q)}$ is an ILFP then the intersection between ILFS and ILFP is defined as $x_{(p,q)} \cap A = \big(p \cap \mu_A(x); q \cup \gamma_A(x)\big)$.

Definition 2.11.[14] An intuitionistic L-topology (ILT for short) on X is a family τ of ILFSs in X which satisfies the following conditions:
(i) $0_{\sim}, 1_{\sim} \in \tau$.
(ii) If $A_1, A_2 \in \tau$ then $A_1 \cap A_2 \in \tau$.
(iii) If $A_i \in \tau$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} A_i \in \tau$.
Then the pair (X, τ) is called an intuitionistic L-topological space (ILTS, for short) and the members of τ are called intuitionistic L-fuzzy open sets (ILFOS for short). An intuitionistic L-fuzzy set B is called an intuitionistic L-fuzzy closed set (ILFC for short) if $1 - B \in \tau$.

Definition 2.12. [9] Let (X, τ) and (Y, s) be two ILTSs. Then a map $f: X \to Y$ is said to be
(i) Continuous if $f^{-1}(B)$ is an ILFOS of X for each ILFOS B of Y, or equivalently, $f^{-1}(B)$ is an ILFCS of X for each ILFCS B of Y,
(ii) Open if $f(A)$ is an ILFOS of Y for each ILFOS A of X,
(iii) Closed if $f(A)$ is an ILFCS of Y for each ILFCS A of X,
(iv) A homeomorphism if f is bijective, continuous and open.

3. Definition and properties of intuitionistic lattice fuzzy R_0 spaces

In this section, we give sixnotions ofR_0 spacein intuitionistic L-topological spaces and establish some of their related theorems.

Definition 3.1. An ILTS (X,τ) is called

(a) $IL-R_0(i)$ if for all $x,y\in X, x\neq y$, whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $\mu_A(x)=1,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)=1$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that and $\mu_B(y)=1,\gamma_B(y)=0,\mu_B(x)=0,\gamma_B(x)=1$.

(b) $IL-R_0(ii)$ if for any pair of distinct ILFP $x_{(p,q)}$, $y_{(r,s)}\in S(X)$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $x_{(p,q)}\in A,\ y_{(r,s)}\notin A$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that $y_{(r,s)}\in B,\ x_{(p,q)}\notin B$.

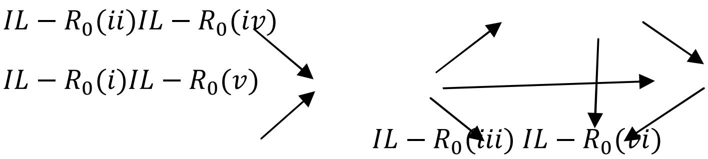
(c) $IL-R_0(iii)$ if for any pair of distinct ILFP $x_{(p,q)}$, $y_{(r,s)}\in S(X)$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)$ with $x_{(p,q)}\in A,\ y_{(r,s)}\cap A=0_\sim$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that $y_{(r,s)}\in B,\ x_{(p,q)}\cap B=0_\sim$.

(d) $IL-R_0(iv)$ if for all $x,y\in X, x\neq y$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $\mu_A(x)>0,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)>0$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that$\mu_B(y)>0,\gamma_B(y)=0,\mu_B(x)=0,\gamma_B(x)>0$.

(e) $IL-R_0(v)$ if for all $x,y\in X, x\neq y$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $\mu_A(x)>\mu_A(y);\gamma_A(y)>\gamma_A(x)$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_B(y)>\mu_B(x);\gamma_B(x)>\gamma_B(y)$.

(f) $IL-R_0(vi)$ if for all $x,y\in X, x\neq y$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $\mu_A(x)\neq\mu_A(y);\gamma_A(x)\neq\gamma_A(y)$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_B(x)\neq\mu_B(y);\gamma_B(x)\neq\gamma_B(y)$

Theorem 3.2. Let (X,τ) be an ILTS. Then we have the following implications:



Proof: $\boldsymbol{IL-R_0(i)\Rightarrow IL-R_0(iv)\Rightarrow IL-R_0(v)\ \Rightarrow\ IL-R_0(vi)}$: Suppose (X,τ) is an $IL-R_0(i)$. Then we have by definition, if for all $x,y\in X, x\neq y$, whenever $\exists$ ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $\mu_A(x)=1,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)=1$ then $\exists B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_B(y)=1,\gamma_B(y)=0,\mu_B(x)=0,\gamma_B(x)=1$. Hence we have

(1) … $\begin{cases}\exists \text{ ILOS } A=(\mu_A,\gamma_A)\in\tau \text{ with } \mu_A(x)>0,\gamma_A(x)=0;\mu_A(y)=0,\gamma_A(y)>0\\ \text{then } \exists\ B=(\mu_B,\gamma_B)\in\tau \text{ such that } \mu_B(y)>0,\gamma_B(y)=0,\mu_B(x)=0,\gamma_B(x)>0\end{cases}$

(2) … … … $\Rightarrow\begin{cases}\exists \text{ ILOS } A=(\mu_A,\gamma_A)\in\tau \text{ with } \mu_A(x)>\mu_A(y);\gamma_A(y)>\gamma_A(x)\\ \text{then } \exists\ B=(\mu_B,\gamma_B)\in\tau \text{ such that } \mu_B(y)>\mu_B(x);\gamma_B(x)>\gamma_B(y).\end{cases}$

(3) … … … $\Rightarrow\begin{cases}\exists \text{ ILOS } A=(\mu_A,\gamma_A)\in\tau \text{ with } \mu_A(x)\neq\mu_A(y);\gamma_A(x)\neq\gamma_A(y)\\ \text{then } \exists\ B=(\mu_B,\gamma_B)\in\tau \text{ such that } \mu_B(x)\neq\mu_B(y);\gamma_B(x)\neq\gamma_B(y).\end{cases}$

From (1), (2) and (3) we see that $IL-R_0(i)\Rightarrow IL-R_0(iv)\Rightarrow IL-R_0(v)\Rightarrow IL-R_0(vi)$.

$\boldsymbol{IL-R_0(i) \Rightarrow IL-R_0(v)}$ **and** $\boldsymbol{IL-R_0(i) \Rightarrow IL-R_0(vi)}$: Suppose (X,τ) is an $IL-R_0(i)$. Then we have by definition, if for all $x,y \in X, x \neq y$, whenever $\exists$ ILOS $A=(\mu_A,\gamma_A) \in \tau$ with $\mu_A(x)=1, \gamma_A(x)=0; \mu_A(y)=0, \gamma_A(y)=1$ then $\exists B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_B(y)=1, \gamma_B(y)=0, \mu_B(x)=0, \gamma_B(x)=1$.

$$(4) \ldots\ldots\ldots \Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } \mu_A(x) > \mu_A(y); \gamma_A(y) > \gamma_A(x) \\ \text{then } \exists\, B=(\mu_B,\gamma_B) \in \tau \text{ such that } \mu_B(y) > \mu_B(x); \gamma_B(x) > \gamma_B(y). \end{cases}$$

$$(5) \ldots\ldots\ldots \Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } \mu_A(x) \neq \mu_A(y); \gamma_A(x) \neq \gamma_A(y) \\ \text{then } \exists\, B=(\mu_B,\gamma_B) \in \tau \text{ such that } \mu_B(x) \neq \mu_B(y); \gamma_B(x) \neq \gamma_B(y). \end{cases}$$

From (4) and (5) we see that $IL-R_0(i) \Rightarrow IL-R_0(v)$and $IL-R_0(i) \Rightarrow IL-R_0(vi)$.

$\boldsymbol{IL-R_0(iv) \Rightarrow IL-R_0(v)}$: Suppose (X,τ) is an $IL-R_0(iv)$. Then we have by definition, if for all $x,y \in X, x \neq y$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A) \in \tau$ with $\mu_A(x) > 0, \gamma_A(x)=0; \mu_A(y)=0, \gamma_A(y)>0$ then $\exists B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_B(y) > 0, \gamma_B(y)=0, \mu_B(x)=0, \gamma_B(x)>0$.

$$(6) \ldots\ldots\ldots \Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } \mu_A(x) > \mu_A(y); \gamma_A(y) > \gamma_A(x) \\ \text{then } \exists\, B=(\mu_B,\gamma_B) \in \tau \text{ such that } \mu_B(y) > \mu_B(x); \gamma_B(x) > \gamma_B(y). \end{cases}$$

This is $IL-R_0(v)$.

$\boldsymbol{IL-R_0(iv) \Rightarrow IL-R_0(vi)}$: Suppose (X,τ) is an $IL-R_0(iv)$. Then we have by definition, if for all $x,y \in X, x \neq y$ whenever $\exists$ ILOS $A=(\mu_A,\gamma_A) \in \tau$ with $\mu_A(x) > 0, \gamma_A(x)=0; \mu_A(y)=0, \gamma_A(y)>0$ then $\exists B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_B(y) > 0, \gamma_B(y)=0, \mu_B(x)=0, \gamma_B(x)>0$.

$$(7) \ldots\ldots\ldots \Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } \mu_A(x) \neq \mu_A(y); \gamma_A(x) \neq \gamma_A(y) \\ \text{then } \exists\, B=(\mu_B,\gamma_B) \in \tau \text{ such that } \mu_B(x) \neq \mu_B(y); \gamma_B(x) \neq \gamma_B(y). \end{cases}$$

From (7) we see that $IL-R_0(iv) \Rightarrow IL-R_0(vi)$.

$\boldsymbol{IL-R_0(ii) \Rightarrow IL-R_0(i)}$: Suppose (X,τ) is an $IL-R_0(ii)$. Then we have for any pair of distinct ILFP $x_{(p,q)}, y_{(r,s)} \in S(X)$, whenever $\exists$ ILOS $A=(\mu_A,\gamma_A) \in \tau$ with $x_{(p,q)} \in A$, $y_{(r,s)} \notin A$ then $\exists B=(\mu_B,\gamma_B) \in \tau$ such that $y_{(r,s)} \in B$, $x_{(p,q)} \notin B$.

$$\Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } p \leq \mu_A(x), q \geq \gamma_A(x); r \geq \mu_A(y), s \leq \gamma_A(y) \\ \text{then } \exists B=(\mu_B,\gamma_B) \in \tau \text{ such that} r \leq \mu_B(y), s \geq \gamma_B(y); p \geq \mu_B(x), q \leq \gamma_B(x) \end{cases}$$

$$\Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } \mu_A(x)=1, \gamma_A(x)=0; \mu_A(y)=0, \gamma_A(y)=1 \text{ and} \\ \text{then } \exists B=(\mu_B,\gamma_B) \in \tau \text{ such that} \mu_B(y)=1, \gamma_B(y)=0; \mu_B(x)=0, \gamma_B(x)=1. \end{cases}$$

As $p,q,r,s \in L=\{0,0.1,0.2,\ldots,1\}$. Which is $IL-R_0(ii) \Rightarrow IL-R_0(i)$.

$\boldsymbol{IL-R_0(iii) \Rightarrow IL-R_0(i)}$: Suppose (X,τ) is an $IL-R_0(iii)$. Then we have by definition, for any pair of distinct ILFP $x_{(p,q)}, y_{(r,s)} \in S(X)$, whenever $\exists$ ILOS $A=(\mu_A,\gamma_A) \in \tau$ with $x_{(p,q)} \in A, y_{(r,s)} \cap A = 0_\sim$ then $\exists B=(\mu_B,\gamma_B) \in \tau$ such that $y_{(r,s)} \in B$, $x_{(p,q)} \cap B = 0_\sim$.

$$\Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } p \leq \mu_A(x), q \geq \gamma_A(x); r \cap \mu_A(y)=0, s \cup \gamma_A(y)=1 \\ \quad \text{and then } \exists B=(\mu_B,\gamma_B) \in \tau \text{ such that} \\ \quad r \leq \mu_B(y), s \geq \gamma_B(y); p \cap \mu_B(x)=0, q \cup \gamma_B(x)=1 \end{cases}$$

$$\Rightarrow \begin{cases} \exists \text{ ILOS } A=(\mu_A,\gamma_A) \in \tau \text{ with } \mu_A(x)=1, \gamma_A(x)=0; \mu_A(y)=0, \gamma_A(y)=1 \text{ and} \\ \text{then } \exists B=(\mu_B,\gamma_B) \in \tau \text{ such that } \mu_B(y)=1, \gamma_B(y)=0; \mu_B(x)=0, \gamma_B(x)=1. \end{cases}$$

As $p,q,r,s \in L=\{0,0.1,0.2,\ldots,1\}$ which is $IL-R_0(iii) \Rightarrow IL-R_0(i)$.

None of the reverse implications is true in general which can be seen from the following counter examples:

Example 3.2.1. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 1,0), (y, 0,1)\}$ and $B = \{(x, 0,1), (y, 1,0)\}$. Hence we see that (X, τ) is an $IL - R_0(i)$ but not $IL - R_0(ii)$and $IL - R_0(iii)$.

Example 3.2.2. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 0.4,0.4), (y, 0.3,0.5)\}$ and $B = \{(x, 0.5,0.4), (y, 0.4,0.5)\}$. Hence we see that (X, τ) is an $IL - R_0(vi)$ but not $IL - R_0(i), IL - R_0(iv)$ and $IL - R_0(v)$.

Example 3.2.3. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 0.5,0.4), (y, 0.4,0.5)\}$ and $B = \{(x, 0.4,0.5), (y, 0.6,0.4)\}$. Hence we see that (X, τ) is an $IL - R_0(v)$ but not $IL - R_0(i)$ and $IL - R_0(iv)$.

Example 3.2.4. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 0.4,0), (y, 0,0.5)\}$ and $B = \{(x, 0,0.5), (y, 0.5,0)\}$. Hence we see that (X, τ) is an $IL - R_0(iv)$ but not $IL - R_0(i)$.
Now we discuss ‘hereditary’ property of ILF –R_0(j) concepts, where (j = i, ii, iii, iv,v, vi.)

Definition 3.3. [14] Let (X, τ) be an ILTS and $A \subseteq X$. we define $\tau_A = \{u|A: u \in \tau\}$ the subspace ILT’s on A induced by τ.Then(A, τ_A) is called the subspace of (X, τ) with the underlying set A.
An IL-topological property ‘P’ is called hereditary if each subspace of an IL-topological space with property ‘P’ also has property ‘P’.

Theorem 3.4. Let (X, τ) be an ILTS and $U \subseteq X$ and $\tau_U = \{A|U: A \in \tau\}$. Then
(a) (X, τ) is $IL - R_0(i) \Longrightarrow (U, \tau_U)$ is $IL - R_0(i)$.
(b) (X, τ) is $IL - R_0(ii) \Longrightarrow (U, \tau_U)$ is $IL - R_0(ii)$.
(c) (X, τ) is $IL - R_0(iii) \Longrightarrow (U, \tau_U)$ is $IL - R_0(iii)$.
(d) (X, τ) is $IL - R_0(iv) \Longrightarrow (U, \tau_U)$ is $IL - R_0(iv)$.
(e) (X, τ) is $IL - R_0(v) \Longrightarrow (U, \tau_U)$ is $IL - R_0(v)$.
(f) (X, τ) is $IL - R_0(vi) \Longrightarrow (U, \tau_U)$ is $IL - R_0(vi)$.
Proof: We prove only (a). Suppose (X, τ) is $IL - R_0(i)$, we prove that (U, τ_U) is $IL - R_0(i)$.Let $x, y \in U, x \neq y$. Then $x, y \in X, x \neq y$ as $U \subseteq X$. Since (X, τ) is $IL - R_0(i)$, we have for all $x, y \in X, x \neq y,$ whenever $\exists$ ILOS $A = (\mu_A, \gamma_A) \in \tau$ with $\mu_A(x) = 1, \gamma_A(x) = 0; \mu_A(y) = 0, \gamma_A(y) = 1$ then $\exists B = (\mu_B, \gamma_B) \in \tau$ such that and $\mu_B(y) = 1, \gamma_B(y) = 0, \mu_B(x) = 0, \gamma_B(x) = 1$.For $U \subseteq X$, we find ILOS $A|U = (\mu_{A|U}, \gamma_{A|U}) \in \tau_U$ with $\mu_{A|U}(x) = 1, \gamma_{A|U}(x) = 0, \mu_{A|U}(y) = 0, \gamma_{A|U}(y) = 1$then $\exists\ B|U = (\mu_{B|U}, \gamma_{B|U}) \in \tau_U$ such that $\mu_{B|U}(y) = 1, \gamma_{B|U}(y) = 0, \mu_{B|U}(x) = 0, \gamma_{B|U}(x) = 1$ as $U \subseteq X$. Hence (U, τ_U) is $IL - R_0(i)$. Similarly (b), (c), (d), (e), (f) can be proved.
We observe here that ILF-R_0(j) ,(j = i, ii, iii, iv, v, vi) concepts are preserved under continuous, one-one and open maps.

Theorem 3.5. Let (X,τ) and (Y,s) be two ILTS, $f:(X,\tau)\to(Y,s)$ be one-one, onto and continuous map. Then

(a) (X,τ) is $IL-R_0(i) \Leftrightarrow (Y,s)$ is $IL-R_0(i)$
(b) (X,τ) is $IL-R_0(ii) \Leftrightarrow (Y,s)$ is $IL-R_0(ii)$
(c) (X,τ) is $IL-R_0(iii) \Leftrightarrow (Y,s)$ is $IL-R_0(iii)$
(d) (X,τ) is $IL-R_0(iv) \Leftrightarrow (Y,s)$ is $IL-R_0(iv)$
(e) (X,τ) is $IL-R_0(v) \Leftrightarrow (Y,s)$ is $IL-R_0(v)$
(f) (X,τ) is $IL-R_0(vi) \Leftrightarrow (Y,s)$ is $IL-R_0(vi)$.

Proof: We prove only (a). Suppose (X,τ) is $IL-R_0(i)$, we prove that (Y,s) is $IL-R_0(i)$. Let $y_1,y_2\in Y$ with $y_1\neq y_2$. Since f is onto, $\exists\, x_1,x_2\in X$, such that $f(x_1)=y_1, f(x_2)=y_2$ and $x_1\neq x_2$ as f is one-one. Again since (X,τ) is $IL-R_0(i)$, we have for all $x_1,x_2\in X, x_1\neq x_2$, whenever $\exists$ an ILOS $A=(\mu_A,\gamma_A)\in\tau$ with $\mu_A(x_1)=1,\gamma_A(x_1)=0,\mu_A(x_2)=0,\gamma_A(x_2)=1$then $\exists\, B=(\mu_B,\gamma_B)\in\tau$ such that $\mu_B(x_2)=1,\gamma_B(x_2)=0,\mu_B(x_1)=0,\gamma_B(x_1)=1$.Since$f:(X,\tau)\to(Y,s)$,whenever $\exists$ ILOS $f(A)=\big(f(\mu_A),1-f(1-\gamma_A)\big)$ with $f(\mu_A)(y_1)=\{\sup\mu_A(x_1): f(x_1)=y_1\}=1$
$\{1-f(1-\gamma_A)\}(y_1)=1-f(1-\gamma_A)(y_1)=1-\{\sup(1-\gamma_A)(x_1): f(x_1)=y_1\}$
$=1-\{\sup\big(1-\gamma_A(x_1)\big): f(x_1)=y_1\}=1-\{\sup(1-0)\}=1-1=0$ And

$$f(\mu_A)(y_2)=\{\sup\mu_A(x_2): f(x_2)=y_2\}=0$$

$\{1-f(1-\gamma_A)\}(y_2)=1-f(1-\gamma_A)(y_2)=1-\{\sup(1-\gamma_A)(x_2): f(x_2)=y_2\}$
$=1-\{\sup\big(1-\gamma_A(x_2)\big): f(x_2)=y_2\}=1-\{\sup(1-1)\}=1-0=1.$ Then$\exists f(B)=(f(\mu_B),1-f(1-\gamma_B))\in s$ such that $f(\mu_B)(y_2)=1;\{1-f(1-\gamma_B)\}(y_2)=0\,; f(\mu_B)(y_1)=0;\{1-f(1-\gamma_B)\}(y_1)=1$.Hence$(Y,s)$ is $IL-R_0(i)$.

Conversely suppose that (Y,s) is $IL-R_0(i)$.We prove that (X,τ) is $IL-R_0(i)$. Let $x_1,x_2\in X$with$x_1\neq x_2\Longrightarrow f(x_1)\neq f(x_2)asf$ is one-one. Put $f(x_1)=y_1,$and $f(x_2)=y_2$then $y_1\neq y_2$. Since (Y,s) is $IL-R_0(i)$, whenever$\exists$ ILOS $A=(\mu_A,\gamma_A)\in s$ with $\mu_A(y_1)=1,\gamma_A(y_1)=0;\ \mu_A(y_2)=0,\gamma_A(y_2)=1$then $\exists\, B=(\mu_B,\gamma_B)\in s$ such that $\mu_B(y_1)=0,\gamma_B(y_1)=1;\ \mu_B(y_2)=1,\gamma_B(y_2)=0.\, i.e.$

$$\begin{cases}\exists \text{ ILOS } A=(\mu_A,\gamma_A)\in s \text{ with} \mu_A f(x_1)=1,\gamma_A f(x_1)=0;\mu_A f(x_2)=0,\gamma_A f(x_2)=1\\ \text{then } \exists\, B=(\mu_B,\gamma_B)\in s\ \mu_B f(x_1)=0,\gamma_B f(x_1)=1;\mu_B f(x_2)=1,\gamma_B f(x_2)=0.\end{cases}$$

$$\Rightarrow\begin{cases}f^{-1}\mu_A(x_1)=1,f^{-1}\gamma_A(x_1)=0;f^{-1}\mu_A(x_2)=0,f^{-1}\gamma_A(x_2)=1 \text{ and}\\ f^{-1}\mu_B(x_1)=0,f^{-1}\gamma_B(x_1)=1;f^{-1}\mu_B(x_2)=1,f^{-1}\gamma_B(x_2)=0.\end{cases}$$

Since $A=(\mu_A,\gamma_A),B=(\mu_B,\gamma_B)\in s,$ Hence it is clear that if $\forall\, x_1,x_2\in X, x_1\neq x_2$ whenever $\exists f^{-1}(A)=(f^{-1}(\mu_A),f^{-1}(\gamma_A))\in\tau$ with $f^{-1}\mu_A(x_1)=1,f^{-1}\gamma_A(x_1)=0;f^{-1}\mu_A(x_2)=0,f^{-1}\gamma_A(x_2)=1$ then $\exists f^{-1}(B)=(f^{-1}(\mu_B),f^{-1}(\gamma_B))\in\tau$ such that $f^{-1}\mu_B(x_1)=0,f^{-1}\gamma_B(x_1)=1;f^{-1}\mu_B(x_2)=1,f^{-1}\gamma_B(x_2)=0.$

Hence (X,τ) is also $IL-R_0(i)$.Similarly, (b), (c), (d), (e), (f) can be proved.

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