Total Pages—10 C/17/DDE/M.Sc./Part-II/Maths./8

2016

M.Sc. Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VIII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Illustrate the answers wherever necessary.

Group—A

Answer Q. No. *1* is compulsory and any *three* from the rest.

1. Answer any *two* of the following : 2×4

(a) Define exponential Fourier transform and state when this transform exists. Also, state its inversion formula.

(Turn Over)

(b) Define Sturm-Liouville problem involving bounding value problem. Write its important properties.

(c) Define finite Hankel transform of order n of a function f(r), $0 \le r \le a$ and state its inversion formula. Find the zero-order Hankel transform of e^{-ar}, $a > 0$ in simplest form.

2. (a) Find the resolvent kernel of the integral equation

$$y(x) = (1 + x) + \lambda \int_0^x (x - t) y(t) dt$$

and the solve it. If $\lambda = 1$, then what happens about the solution of given integral equation. 7

(b) Find the solution of the following problem of free vibration of a stretched string of infinite length

$$\text{DDE} : \frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0, -\infty < x < \infty$$

BCS : u (x, 0) = f(x)

$$\frac{\partial}{\partial t} u(x, 0) = g(x)$$

u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 7

3. (a) Find the exponential fourier transform of f(t) where

$$f(t) = \begin{cases} 1 - |t|, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}.$$

Hence evaluate $\int_0^\infty \frac{1 - \cos w}{w^2} dw$. 7

(b) Using Green's function, solve the boundary value problems

$$\frac{d^2 y}{dx^2} - y = x, \quad y(0) = y(1) = 0.$$ 7

4. (a) Show that the eigen functions of Starm-Liouville problem in a closed interval are orthogonal with respect to the weight function in that closed interval. 6

(b) Prove that

$$H_n\left\{\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{n^2}{r^2} f\right\} = -\alpha^2 f_n(\alpha)$$

provided both rf′(r) and rf(r) tend to zero as $r \to 0$ and $r \to \infty$, where H_n stands for nth order Hankel transform. 5

(c) Define regular sequence. When two regular sequences are equivalent ? What is generalised function ? 3

5. (a) State and prove the convolution theorem of Laplace transform and hence find the function whose Laplace transform is $\frac{p}{(p^2+a^2)^2}$. 7

(b) Reduce the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda xy = 1, \ 0 \le x \le \ell$$

with boundary conditions $y(0) = 0$, $y(\ell) = 1$ to an integral equation and find kernel. 7

6. (a) Discuss the solution procedure of homogeneous Fredholm integral equation of the second kind with degenerate kernel. 6

(b) If a function $\frac{f(t)}{t}$ satisfies the conditions of its Laplace transform and $L\{f(t)\} = F(p)$, which exists for real $(p) > \gamma$, then prove that

$$L\left\{\frac{f(t)}{t}\right\} = \int_p^\infty F(u)du,$$

where γ is an exponential order. 5

(c) Find the zero-order Hankel transform of the function $H(q-r)$, where $H(r)$ is Heaviside's step function. 3

Group—B

(Elements of Optimization and Operations Research)

[Marks : 50]

Answer Q. No. 7 and any *three* from the rest.

7. State Bellman's "principle of optimality" related to dynamic programming. 2

or

What do you mean by the term "Economic order quantity". 2

8. (a) Use dynamic programming to solve the problem

$$\text{Max. } Z = x_1^2 + x_2^2 + x_3^2$$

subject to $x_1.x_2.x_3 = 6$

$x_1, x_2, x_3 \ge 0$ 8

(b) Find the range of discrete change of requirement vectors of the LPP

Max. $Z = CX$

subject to $AX = b$

$x \ge 0$

so that the feasibility remains undisturbed. 8

9. (a) Solve the following LPP by revised simplex method

$$\text{Max. } Z = 2x_1 - x_2$$

subject to $x_1 + x_2 \le 4$

$2x_1 - x_2 \ge 20$

$x_1, x_2 \ge 0$ 8

(b) Write the steps of Wolfe's method to solve a quadratic problem. 8

10. (a) Solve the following LPP by Gomory's cutting plane method

$$\text{Max. } Z = x_1 + x_2$$

subject to $3x_1 + 2x_2 \le 12$

$x_2 \le 2$

$x_1, x_2 \ge 0$ and integers. 8

(b) Find the economic ordering quantity for inventory model with instantaneous replemishment, uniform finite demand and zero lead time. Assumed that shortages are permitted. 8

11. (a) Solve the following quadratic problem by Beale's method

$$\text{Max. } Z = 2x_1 + 3x_2 - x_1^2$$

subject to $x_1 + 2x_2 \le 4$

and $x_1, x_2 \ge 0$ 8

(b) The optimal table of the LPP

$$\text{Max. } Z = 15x_1 + 45x_2$$

subject to $x_1 + 16x_2 \le 240$

$5x_1 + 2x_2 \le 162$

$x_2 \le 50$

and $x_1, x_2 \ge 0$

is found to be

C_B	Basis	X_B	y_1	y_2	y_3	y_4	y_5
45	x_2	$\frac{173}{13}$	0	1	$\frac{5}{78}$	$\frac{1}{78}$	0
15	x_1	$\frac{352}{13}$	1	0	$-\frac{1}{39}$	$\frac{8}{39}$	0
0	x_5	$\frac{477}{13}$	0	0	$-\frac{5}{78}$	$\frac{1}{78}$	1

Find the range of the values of C_1 and C_2 for which the current optimal solution undisturbed when change one at a time. 8

12. (a) A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amounts to Rs. 0.60 per unit per year. The set up cost per run is Rs. 80.00. Find the optimum run size and the minimum average yearly cost. 7

(b) State the sufficient conditions of optimality of a multi-variable function with equality constraints. Solve the following problem by using the method of Lagrangian Multipliers

$$\text{Min } z = x_1^2 + x_2^2 + x_3^2$$

subject to the constraints

$$x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$
$$\text{and } x_1, x_2, x_3 \geq 0$$

3+6

Group—B

(Dynamical Oceanology and Meteorology)

[Marks : 50]

Answer Q. No. *12* and any *three* from the rest.

7. (a) Derive the area equivalence property of tephigram and discuss its important features. 8

(b) Define specific entropy. Establish the relation between the specific entropy and the potential temperature. 5

(c) Explain the convergence and divergence in the atmosphere. 3

8. (a) How is a gradient wind generated in the atmosphere ? Discuss different cases of its occurrences. 8

(b) Derive the angle between the frontal surface and earth's surface in the atmosphere. 5

(c) What is the concept of coriolis force in the atmosphere ? 3

9. (a) Find the rate of change circulation in the atmosphere and interpret each temp. 8

(b) What do you mean by adiabatic process ? Deduce the Poisson's equation in the following form

$$\frac{T}{\theta} = \left(\frac{p}{1000}\right)^{\frac{R}{C_p}}.$$ 4

(c) Define the potential temperature and show that it is invariant during the adiabatic motion in the atmosphere. 4

10. (a) Deduce Gibb's general thermodynamical relation for sea water. Hence, derive Gibb's-Duhem relation.

(b) Derive the equations of conservation of mass of sea water taken as mixture of two components of salt and pure water. 5+5+6

11. (a) Show that equation of motion of sea water can be expressed as

$$\frac{D\vec{q}}{Dt} = \vec{F} + 2\vec{q} \times \vec{\Omega} - \frac{1}{\rho}\vec{\nabla}p + \frac{\mu}{3\rho}\left[\vec{\nabla}\Theta + 3\nabla^2\vec{q}\right]$$

(symbols have their usual meanings).

(b) Find the condition of stable mechanical equilibrium of stratified sea-water. 12+4

12. Define the virtual temperature. 1×2

or

What do you meant the terms potential temperature and potential density of the system.

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