2016

M.Sc. 4th Seme. Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-404 (OR/OM)

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Non-linear Optimization / Dynamical Oceanology-II*)

MTM-404 (OR)

(Non-linear Optimization)

Answer Q. No. *1* and any *three* from the rest.

1. Answer any *five* from the following : 2×5

(a) What do you mean by quadralic programming problem ? Write the explicit expression of it.

(b) Define monomial and polynomial. Give an example.

(c) What is chance constrained programming technique ?

(d) What do you mean by theorems of the alternative ?

(e) State Karlin's constraint qualification.

(f) Define Bimatrix game with an example.

(g) State Dorn's duality theorem.

(h) Define geometric programming with an example.

2. (a) Solve the quadratic programming problem using Wolfe's method

$$\text{Maximize } z = 2x_1 + 3x_2 - 2x_1^2$$
$$\text{subject to } \quad x_1 + 4x_2 \le 4,$$
$$x_1 + x_2 \le 2,$$
$$x_1, x_2 \ge 0.$$

(b) State and prove Motzkin's theorem of alternative.

7+3

3. (a) Minimize

$$f(x) = \frac{1}{x_1 x_2} + 10 x_1 x_2 x_3^{-} + 20 x_2 x_3 + x_1 x_3$$

$$x_1, x_2, x_3 > c$$

using Geometric programming.

(b) State and prove Weak duality theorem in connection with duality in non-linear programming

6+4

4. (a) Define multi-objective non-linear programming probelm. Also define the Pareto Optimal and Weak Pareto Optimal solutions in connection with this.

(b) State and prove Wolfe's duality theorem.

5+5

5. (a) Find the Nash equalibrium solution(s) of the following bimatrix game (if exists)

$$\begin{bmatrix} (-2, -1) & (1, 1) \\ (-1, 2) & (-1, -2) \end{bmatrix}$$

(b) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. θ is concave on Γ if and only if

$$\theta(x^2) - \theta(x^1) \leqq \theta(x^1)(x^2 - x^1) \text{ for each } x^1, x^2 \in \Gamma.$$

Give the geometrical interpretation of the above result.

3+(5+2)

6. (a) Use the chance constrained programming technique to find an equivalent deterministic LPP to the following Stochastic programming problem.

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i$$

$$x_j \ge 0,\ i, j = 1, 2, \ldots, n$$

When c_j is a random variable.

(b) Define the following :

(i) Minimization problem ;

(ii) Local minimization problem ;

(iii) Kuhn-Tucker stationary point problem ;

(iv) Fritz-John stationary point problem. 4+6

[Internal Assesment : 10 Marks]

MTM-404 (OM)

(Dynamical Oceanology-II)

Answer Q. No. *1* and any *five* from the rest.

1. Answer any *five* from the following : 5×2

(a) Write the vorticity equation for 2D depth averaged shallow water and then interpret the terms.

(b) Derive the depth-integrated continuity equation for shallow water.

(c) What is Backing and veering ? Discuss these based on thermal wind and geostropic wind.

(d) Write the continuity equation in isobaric co-ordinate system and then interpret the terms.

(e) Write the expression for Rossby number and discuss its physical significance for smaller and larger values.

(f) Write the final quasi-geostropic zonal and meridional momentum equations.

(g) Write the assumptions for Inertial waves and then write its governing equations.

2. Derive the expression for vorticity (in terms of shear and curvature vorticity) in natural co-ordinate. 6

3. Write expression for potential vorticity and then derive the equation for conservation of potential vorticity. 6

4. State and prove the Taylor-Proudman theorem. 6

5. Write the horizontal equations of motion interms of angular rotation of Earth and then transform the n-component momentum equation interms of isobaric co-ordinates. 6

6. Split the total horizontal velocity into geostropic and ageostropic components and derive the modified equations of motion in terms of geostropic components stating necessary approximation of quasi geostropic theory. 6

7. Under shallow-water theory, state the necessary assumptions for linear waves in the absence of Earth rotation and write the set of equations for those waves. Finally, derive the expressions for surface elevation and horizontal velocity interms of initial surface elevation $F(\lambda)$.

In deep ocean (H = 4500 m) and shallow ocean (H = 110 m), find the phase speed, where it has its usual meaning. 6

8. Write the set of governing equations for sverdrup waves with necessary assumptions and then derive the expressions for surface elevation (z) and velocity components (u, v, w). Finally show that the horizontal velocity components (u, v) describes an ellipse where the ratio of the major axis to the minor axis is $\left|\frac{w}{f}\right|$ where w is wave frequency and f is coriolis parameter. 6

[Internal Assesment : 10 Marks]
