2016

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-302

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Transform and Integral Equation]

Answer Q. No. *1* is compulsory and any *three* from the rest.

1. Answer any *five* questions from the following : 5×2

(a) If $\bar{f}(k, l)$ be the two-dimensional Fourier transferm of a function f(x, y), then what is the Fourier inversion formula to get f(x, y) from $\bar{f}(k, l)$.

(Turn Over)

(b) What do you mean by Fredholm integral equation ? Give an example for non-homogencous Fredholm integral equation.

(c) Define continuous wavelet function and also explain the inverse wavelet transform.

(d) Show that $L\left[\int_0^t f(\tau)d\tau\right] = \frac{F(p)}{p}$, where F(p) is the Laplace transform of f(t).

(e) Prove that the convolution operations for Laplace transform is commutative.

(f) Define eigen value of eigen function involving an integral equation.

(g) Find the value of f(0) and f′(0), when $\bar{f}(p) = \frac{1}{p(p^2+q^2)}$ using initial value theorem in connection with Laplace transform.

2. (a) Discuss the solution procedure of homogeneous fredholm integral equation of the second kind with degenerate kernal. 6

(b) Using Laplace transform find the solution of the equation

$$\frac{d^4x}{dt^4} + 2\frac{d^2x}{dt^2} + x = \sin t$$

satisfying the initial conditions,

$$x(0) = x'(0) = x''(0) = x'''(0) = 0.$$ 6

3. (a) Find the exponential Fourier transform of f(t)

where $f(t) = \begin{cases} 1 - |t|, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$. 4

(b) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform. 6

4. (a) Form an integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} - (\sin x)\frac{dy}{dx} + e^x y = x$$

with the initial conditions $y(0) = 1$ and $y'(0) = -1$. 5

(b) State and prove the convolution theorem of Laplace transform. Use this theorem to show that

$$L^{-1}\left[\frac{p}{(p^2 + a^2)^2}\right] = \frac{t}{2a}\sin at.$$ 5

5. (a) Find the solution of the following problem of free vibration of a stretched string of an infinite length :

PDE : $\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\,\frac{\partial^2 u}{\partial t^2} = 0, \quad -\infty < x < \infty$

Boundary conditions

$$u(x, 0) = f(x)$$

$$\frac{\partial}{\partial t}\, u(x, 0) = g(x)$$

u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 7

(b) Find the solution of the integral equation

$$\frac{1}{\sqrt{\pi}} \int_0^x \frac{\varphi(t)}{\sqrt{x - t}}\, dt = f(x),$$

by the use of Laplace transform, where f(x) is a given function of x. 3

6. (a) State and prove initial value theorem in respect of Laplace transform. Why it is called initial value theorem ? 5+1

(b) Prove that the Fourier transform of $\frac{1}{x}$ is $i\sqrt{\frac{\pi}{2}}\,\mathrm{sgn}(\alpha)$ where sgn is signum function. 4

(Internal Assessment : 10 Marks)
