2016

M.Sc. 2nd Seme. Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-205

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(General Topology & Fuzzy Sets and Their Applications)

Unit-I

(General Topology)

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Give an example where subspace of a normal space need not be normal justify your answer.

(b) Show that $\mathbb{R}$ (the set of all real numbers) is compact in the finite complement topology.

(c) If $\mathfrak{I}$ and $\mathfrak{I}'$ are topologies on X and $\mathfrak{I}'$ is strictly finer than $\mathfrak{I}$, what can you say about the corresponding subspace topologies on the subset Y of X ?

2. (a) If B is a basis for the topology of X, then show that the collection $B_Y = \{B \cap Y \mid B \in B\}$ is a basis for the subspace topology on Y.

(b) Define interior and closure of a set in a topological space.

(c) Let A be a subset of a topological space X. Then show that $x \in \overline{A}$ if and only if every open set U containing x intersects A. 2+2+4

3. (a) Give an example of a topological space where a sequence can converge more than one point. Justify your answer.

(b) Let $f : X \to Y$ be a function where X and Y are topological spaces. Then show that the following are equivalent :

i. $f^{-1}(F)$ is closed in X for each closed set F in Y,

ii. $A \subseteq X$, $f(\overline{A}) \subseteq \overline{f(A)}$

(c) Let $Y \subset X$ and X, Y be connected space. Show that if A and B form a seperation of $(X - Y)$, then $Y \cup A$ and $Y \cup B$ are connected. 2+4+2

4. (a) Define locally compact and completely regular spaces with example.

(b) Show that every compact Hausdorff space is normal.

(c) Give an example of a space which is first countable but not second countable. Justify your answer. 2+4+2

[Internal Assessment —5]

Unit-II

(Fuzzy sets and Their Applications)

[Marks : 25]

Answer Q. No. *1* and any *three* from the rest.

1. Answer any *one* question : 1×2

(a) Let the universal set X = {1, 2, 3, 4, 5} and the function f(x) = [x] on X. Find $F(\tilde{A})$, where $\tilde{A}$ = {(1, 1), (2, 0.8), (3, 0.5), (4, 0.3), (5, 0.1)}.

(b) State Bellman and Zadeh's principle related to fuzzy optimization.

2. Draw the graph of the membership function of the following fuzzy set $\underset{\sim}{A}$:

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & \text{for } x \le 1 \\ 3(x-1)/8 & \text{for } 1 < x \le 3 \\ (6-x)/4 & \text{for } 3 < x < 4 \\ 1/3 & \text{for } x = 4 \\ (3x-2)/20 & \text{for } 4 < x < 6 \\ 4(7-x)/5 & \text{for } 6 \le x \le 7 \\ 0 & \text{for } x > 7 \end{cases}$$

Is it normal? Find the height. Show that it is not convex. Determine the α-cut when $\alpha = 0.6$. 2+1+1+1+1

3. Define a convex fuzzy set.
Using α-cut prove that
$[a_1, b_1, c_1] - [a_2, b_2, c_2] = [a_1 - c_2, b_1 - b_2, c_1 - a_2]$ 1+5

4. Let the membership functions of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < 1 \\ \dfrac{x-1}{4} & \text{if } 1 \le x < 5 \\ \dfrac{7-x}{2} & \text{if } 5 \le x < 7 \\ 0 & \text{if } x \ge 7 \end{cases}$$

$$\mu_{\tilde{B}}(x) = \begin{cases} 0 & \text{if } x < 5 \\ \frac{x-5}{2} & \text{if } 5 \le x < 7 \\ \frac{10-x}{2} & \text{if } 7 \le x < 10 \\ 0 & \text{if } x \ge 10 \end{cases}$$

Find the membership functions of $\tilde{A}^c$, $\tilde{A} \cup \tilde{B}$ and $\tilde{A} \cap \tilde{B}$.

2+2+2

5. Discuss Verdegay's approach to formulate equivalent crisp LPP for a fuzzy LPP. Using this formulate the crisp LPP equivalent to the fuzzy LPP given below

$$\text{Max } Z = x_1 + 2x_2$$

$$\text{subject to } x_1 \le 4 \text{ to } 6$$

$$x_1 - x_2 \le 2 \text{ to } 3$$

$$x_1, x_2 \ge 0$$

4+2

6. (a) What are the basic differences between werner's approach and Zimmermann's approach to solve a fuzzy LPP ?

(b) Using Zimmermann's method, determine the crisp LPP equivalent to the fuzzy LPP

$$\widetilde{\text{Max}} \quad Z = 13x_1 + 12x_2$$

subject to

$$4x_1 + 3x_2 \leq 12 \text{ to } 13$$

$$2x_1 + 5x_2 \leq 10 \text{ to } 11$$

$$3x_1 + 4x_2 \leq 12 \text{ to } 14$$

$$x_1, x_2 \geq 0$$

Where lower bound of the value of the fuzzy objective function is 25 with tolerance 5. 2+4

[Internal Assessment — 5]
