2016

M.Sc. 2nd Seme. Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-204

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*General Theory of Continuum Mechanics*)

Answer Question No *1* and any *four* from the rest.

1. Answer any *two* questions : 4×2

(a) Show that the difference of the values of a two dimensional stream function at two points represents the flux of a fluid across any curve joing those points.

4

(b) For the displacement field

$$u_1 = x_1^2 x_2,\ u_2 = x_2 - x_3^2,\ u_3 = x_2^2 x_3,$$

determine the unit relative displacement vector at P(1, 2, –1) with respect to Q(4, 2, 3). 4

(c) Define stress quadric of cauchy. Prove that the normal stress across any plane through the centre of stress quadric is equal to the inverse of the square of the central radius vector of the quadric normal to the plane. 4

2. Define principal stress. Prove that the extreme values of normal stresses at a point in a continuum are principal stresses. 8

3. (a) Give the significance of volumetric strain in terms of infinitesimal strain tensors. 4

(b) Show that the following are not possible strain components :

$$E_{11} = K\left(x_1^2 + x_2^2\right),\ E_{22} = K\left(x_2^2 + x_3^2\right),$$

$$E_{12} = K' x_1 x_2 x_3,\ E_{13} = E_{23} = E_{33} = 0.$$

K and K′ being constants. 4

4. If the fluid is bounded on all sides by fixed boundary so that the fluid moves only tangentially over the surface, prove that the sum of its Kinetic, potential and intrinsic energies remains constant with the passage of time. 8

5. An infinite mass of fluid acted on by a force $\mu r^{-3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere r = c in it, show that the cavity will be filled up after an interval of time

$$\left(\frac{2}{5\mu}\right)^{1/2} c^{5/4}.$$ 8

6. State and prove Milne-Thomson's circle theorem. Define image of source. Derive the image of a source with respect to circle by this theorem. 8

[Internal Assessment —10]

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