2016

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-101

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Real Analysis]

Answer Q. No. *1* and any *four* from Q. No. *2* to Q. No. *7*.

1. Answer any *four* questions : 4×2

(a) Define open cover for metric space. Give an open cover for (0, 1).

(b) Give an example of a sequence $\{A_n\}$ of connected subsets of $\mathbb{R}^2$ such that $A_{n+1} \subset A_n$ for $n \in \mathbb{N}$, but $\bigcap_{n \in \mathbb{N}} A_n$ is not connected.

(c) Show that a closed subset of a compact metric space is compact.

(d) Let $s_1, s_2 \in \pi_0^+$. Show that if $s_1 \geq s_2$ then

$$\int s_1 d\mu \geq \int s_2 d\mu .$$

(e) Define Borel set.

2. (a) Let $f : [a, b] \to R$ be continuous and

$$F(x) = \int_a^x f(t)dt, \ a \leq x \leq b .$$

Prove that F(x) is a function of bounded variation over [a, b]. Also find the total variation of F(x) over [a, b].

(b) Let $f : [0, 1] \to R$ be defined as

$$f(x) = 2x \sin\frac{\pi}{x} \quad \text{if} \quad 0 < x \leq 1$$
$$= 0 \quad \text{if} \quad x = 0$$

Show that f(x) is not of bounded variation over [0, 1].

5+3

3. (a) Let $\{A_k\}$ be an increasing sequence of measurable sets, such that $\bigcup_{k=1}^{\infty} A_k = A$ is bounded. Prove that A is measurable and $m(A) = \lim_{k \to \infty} m(A_k)$.

(b) Prove that a bounded set S of reals is measurable if and only if its characteristic function is a measurable function. 4+4

4. (a) State and prove the First Mean-value theorem.

(b) Let $f : [a, b] \to \mathbb{R}$ be continuous and $\alpha : [a, b] \to \mathbb{R}$ be monotonically increasing on $[a, b]$. Then show that $f \in R(\alpha)$. 4+4

5. (a) Let u and v be real measurable functions on a measurable space X, also let $\phi : \mathbb{R}^2 \to Y$ be a continuous mapping and $h(x) = \phi(u(x), v(x))$ for $x \in X$, where Y is a metric space. Then prove that the function $h : x \to Y$ is measurable.

(b) State and prove that Lebesgue's Monotone convergence theorem. 4+4

6. (a) Let $\in > 0$ and $f \in L'(\mu)$. Then prove the following Chebyshev's inequality :

$$\mu\{x \in x|\ |f(x)| \geq \epsilon\} \leq \frac{1}{\epsilon} \int |f| du \quad C + \alpha$$

(b) Give an example of a function which is not Riemann integrable but Lebesgue integrable.

(c) State the Egoroff's thorem. 3+3+2

7. (a) Define the Lebesgue integration of an unbounded function $f : E \to R$. Explain geometrically how the definition differs from Riemann integration.

(b) Evaluate the Lebesgue integral

$\int_0^1 f(x)dx$ where

$$f(x) = \begin{cases} \dfrac{3}{5.x^{5/6}}, & 0 < x \le 1 \\ 0 \quad, & x = 0 \end{cases}$$

3+5

(Internal Assessment : 10 Marks)
