*Journal of Physical Sciences, Vol. 21, 2016, 47-61*
*ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal*
*Published on 24 December 2016*

# Multi-Stage Transportation Problem under Vehicles

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*Received 15 September 2016; accepted 3 November2016*

**ABSTRACT**

In this paper, the problem formulation, model development and solution methodology of cost varying transportation problem(CVTP) is developed. The mathematical models of multistage transportation problem(MSTP) and cost varying multistage transportation problem(CVMSTP) are formulated. Our aim is to solve CVMSTP by solution methodology of CVTP. Numerical example are presented to illustrate the problem formulation, initial allocation and optimality test for both CVTP and CVMSTP.

***Keywords:*** Transportation, Cost Varying Transportation Problem, Multi-stage Transportation Problem, Bi-level Mathematical Programming

## 1. Introduction

Transportation problem(TP) [1,2,8,11] is a special class of linear programming problem. This problem deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. For this problem the following information are to be needed:

(P1) Available amount of the commodity at different origins.

(P2) Amounts demanded at different destinations.

(P3) The transportation cost of one unit of commodity from various origin to various destination.

Transportation Problem is studied by Hitchcock [9] in 1941, and then separately by Koopmans [12] in 1947 and finally placed in the framework of Linear Programming and solved by simplex method by Dantzing [10] in 1951. Since then, improved models solution methods have been developed and the range of application have been steadily widened.

The unit transportation cost is not constant. The ambiguity is found in pre-paid transportation system in our daily life. To transport quantities we contract a vehicle in a fixed charge for a single trip. Different types of vehicles should be contracted in different charges depending on the carrying capacity. So depending on the vehicles the unit transportation cost is varied. From this angle of view, (P3) is changed as the transportation cost of unit quantity is not constant where as the cost of single trip of each vehicle is constant which depends on its carrying capacity. This type of TP is named as CVTP.

In TP it is seen that the solution of TP is depended on its initial basic feasible solution(IBFS) and optimality test. In TP, the IBFS are determine by any one of North West Corner Method(NWCM) or Row Minimum Method(RMM) or Column Minimum Method(CMM) or Matrix Minimum Method(MMM) or Vogel's Approximation Metod(VAM). The methods (except NWCM) are fully depend on unit transportation cost. To determine the IBFS for CVTP Panda and Das [3, 4, 5, 6, 7] have used NWCM. In this paper, other techniques have to be described to determine IBFS for CVTP. The modification of the methods are described by fixed charges of vehicles. Since optimality test depend on unit transportation cost, so unit transportation cost for CVTP are determined by Das and Panda's [3, 4, 5, 6, 7] proposed algorithm. It is notated that the unit transportation cost may changed during optimality test.

Multistage transportation problem is a special call of TP in which quantities are transported from supplers to distributer, distributers, to retailers, retailers to purchasers. In this paper we want to adopt the concept of finite CVTP in multistage transportation problem to develop mathematical model and solution methodology for multistage transportation problem under finite vehicles.

## 2. Mathematical formulation

A classical TP is formulated in following mathematical model

**Model 1**

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\textit{subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,\ldots,m \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ \ j = 1,\ldots,n \tag{2}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geq 0\ \ \forall i,\ \forall j$$

### 2.1. Cost varying transportation problem (CVTP)

Suppose there are $N$ -types off vehicles $V_l, r = 1,\ldots,N$ from each source to each destination. Let $C_r, r = 1,\ldots,N$ are the capacities(in unit) of the vehicles $V_r, r = 1,\ldots,N$ respectively, where $C_1 < C_2 < \ldots < C_N$. So, CVTP can be represent in the following tabulated form.

**Table** 1. Tabular form of CVTP under $N$ -vehicle

| | $D_1$ | $D_2$ | .. | $D_n$ | *stock* |
|---|---|---|---|---|---|
| $O_1$ | $R_{11}^1,.., R_{11}^N$ | $R_{12}^1,.., R_{12}^N$ | .... | $R_{1n}^1,.., R_{1n}^N$ | $a_1$ |
| $O_2$ | | | .... | | $a_2$ |

| | | | | | |
|---|---|---|---|---|---|
| | $R_{21}^1,.,R_{21}^N$ | $R_{22}^1,.,R_{22}^N$ | | $R_{2n}^1,.,R_{2n}^N$ | |
| .... | .... | .... | .... | .... | .... |
| $O_m$ | $R_{m1}^1,.,R_{m1}^N$ | $R_{m2}^1,.,R_{m2}^N$ | .... | $R_{mn}^1,.,R_{mn}^N$ | $a_m$ |
| *Demand* | $b_1$ | $b_2$ | .... | $b_n$ | |

where $a_i$ is the quantity of material available at source $O_i, i = 1,\dots,m$

$b_j$ is the quantity of material required at destination $D_j, j = 1,\dots,n$

$R_{ij} = (R_{ij}^1,\dots,R_{ij}^N)$ represents transportation cost for each cell $(i, j)$; where $R_{ij}^l$ is the transportation cost from source $O_i, i = 1,\dots,m$ to the destination $D_j, j = 1,\dots,n$ by the vehicle $V_l, l = 1,\dots,N$.

$c_{ij}$ is the unit cost of transportation from st source $O_i$ to destination $D_j$.

**Definition 1. Feasible Solution (FS):** A set of non-negative allocations $x_{ij} \geq 0$ which satisfies $(1),(2)$ is known as feasible solution.

**Definition 2. Basic Feasible Solution (BFS):** A feasible solution to a $m$-origin and $n$-destination problem is said to be basic feasible solution if number of positive allocations are $(m+n-1)$.

If the number of allocations in a basic feasible solutions are less than (m+n-1), it is called degenerate basic feasible solution (DBFS) otherwise non-degenerate basic feasible solution (NDBFS).

**Definition 3. Optimal Solution:** A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

**2.1.1. Solution Method CVTP**

The transportation methods for finding an optimum solution to the T.P. consists of the following steps:

- Find an initial basic feasible solution(IBFS)
- Test the optimality of IBFS.
- If the solution is optimal , end the process. Otherwise , improve it by a'jump' to an adjacent basic feasible solution that yields the highest rate of improvement in the value of the objective function.
- Return to step (ii) and repeat the process until an optimal solution has been obtain.

**2.1.2. Determination of IBFS**

To determine the IBFS we apply any one of the following procedure

**North-West corner Method (NWCM)**

**Step 1.** Compute min $(a_1,b_1)$. If $a_1 < b_1,$ min $(a_1,b_1) = a_1$ and if $a_1 > b_1,$ min

$(a_1, b_1) = b_1$.

Select $x_{11}$ = min $(a_1, b_1)$ allocate the value of $x_{11}$ in the cell $(1,1)$.

**Step 2.** If $a_1 < b_1$, compute min $(a_2, b_1 - a_1)$. Select $x_{21}$ = min $(a_2, b_1 - a_1)$ and allocate the value of $x_{21}$ in the cell $(2,1)$.

If $a_1 > b_1$, compute min $(a_1 - b_1, b_2)$. Select $x_{12}$ = min $(a_1 - b_1, b_2)$ and allocate the value of $x_{12}$ in the cell $(1,2)$.

Let us now make an assumption that $a_1 - b_1 < b_2$. With this assumption the next cell for which
some allocation is to made, is the cell $(2,2)$.

If $a_1 = b_1$ then allocate $0$ only in one of two cells $(2,1)$ or $(1,2)$. The next allocation is to be made cell $(2,2)$.

In general, if an allocation is made in the cell $(i+1, j)$ in the current step, the next allocation will be made either in cell $(i, j)$ or $(i, j+1)$.

The feasible solution obtained by this away is always a BFS.

**Example 2.1.**

Consider a CVTP as

| | $D_1$ | $D_2$ | $D_3$ | *stock* |
|---|---|---|---|---|
| $O_1$ | 7,10 | 5,7 | 8,12 | 50 |
| $O_2$ | 6,8 | 9,12 | 7,9 | 40 |
| $O_3$ | 4,6 | 12,15 | 8,13 | 30 |
| *Demand* | 35 | 45 | 40 | |

The capacities of vehicles of $V_1$ and $V_2$ are respectively, $C_1 = 15$ and $C_2 = 25$.
By NWCM, the IBFS **Example 2.1.** is given as follows.

| | $D_1$ | $D_2$ | $D_3$ | *stock* |
|---|---|---|---|---|
| $O_1$ | $x_{11} = 35$ 7,10 | $x_{12} = 15$ 5,7 | 8,12 | 50 |
| $O_2$ | 6,8 | $x_{22} = 30$ 9,12 | $x_{23} = 10$ 7,9 | 40 |
| $O_3$ | 4,6 | 12,15 | $x_{33} = 30$ 8,13 | 30 |
| *Demand* | 35 | 45 | 40 | |

| | $D_1$ | $D_2$ | $D_3$ | *stock* |
|---|---|---|---|---|
| $O_1$ | $x_{11} = 5$ 7,10 | $x_{12} = 45$ 5,7 | 8,12 | 50 |
| $O_2$ | $x_{21} = 30$ 6,8 | 9,12 | $x_{23} = 10$ 7,9 | 40 |
| $O_3$ | 4,6 | 12,15 | $x_{33} = 30$ 8,13 | 30 |
| *Demand* | 35 | 45 | 40 | |

**Modified row-minima method(MRMM)**

In this method, we first consider the first row and find the minimum cost cell. Let $(1,k)$ cell be the cell in the first row with minimum cost entities $(R^1_{l,k}, R^2_{l,k})$. We allot in this cell the maximum allocation, i.e, $x_{1k} = min(a_1, b_k)$ .If $a_1 < b_k$ , then $x_{1k} = a_1$ and we cross out the first row and consider the remaining tableau and proceed in same way. Again if $a_1 > b_k$, then $x_{1k} = b_k$ and we cross out the $k^{th}$ column and consider the remaining row of the tableau and proceed next in the same way. If $a_1 = b_1$,then either 1st row or the $k^{th}$ column will be crossed out and the remaining tableau will be consider.By MRMM, the IBFS.

**Example 2.1.** is given as follows.

**Modified column-minima method(MCMM)**

This method is exactly same as the Row-minima method. In this method, we are to start with first column instead of first row and the successive steps we consider only columns. By MCMM, the IBFS of **Example 2.1** is given as follows.

| | $D_1$ | $D_2$ | $D_3$ | *stock* |
|---|---|---|---|---|
| $O_1$ | 7,10 | $x_{12} = 45$ 5,7 | $x_{13} = 5$ 8,12 | 50 |
| $O_2$ | $x_{21} = 5$ 6,8 | 9,12 | $x_{23} = 35$ 7,9 | 40 |
| $O_3$ | $x_{31} = 30$ 4,6 | 12,15 | 8,13 | 30 |
| *Demand* | 35 | 45 | 40 | |

**Modified matrix-minima method(MMMM)**

This method finds a better starting solution. In this method we first find out the cell with minimum cost entities in the cost matrix and allocate in that cell the maximum allowable amount. We then cross out the satisfied row or column and adjust the amounts of supply and demand accordingly.

We repeat the process with uncrossed out matrix and we are left at the end with exactly one uncrossed out row or column. If the cell with minimum cost is not unique, then any one of these cells may be selected for allotment.

By MCMM, the IBFS of **Example 2.1.** is given as follows.

**Modified Vogel's approximation Method(MVAM)**

| | $D_1$ | $D_2$ | $D_3$ | *stock* |
|---|---|---|---|---|
| $O_1$ | 7,10 | $x_{12} = 45$ 5,7 | $x_{13} = 5$ 8,12 | 50 |
| $O_2$ | $x_{21} = 5$ 6,8 | 9,12 | $x_{23} = 35$ 7,9 | 40 |
| $O_3$ | $x_{31} = 30$ 4,6 | 12,15 | 8,13 | 30 |
| *Demand* | 35 | 45 | 40 | |

In this method the allocation is made on the basis of the opportunity (or penalty or extra) cost entities that would be incurred if allocation in certain cells with minimum unit transportation cost entities were missed. The steps in modified Vogel's approximation method(MVAM) are as follows:

**Step 1.** Calculate the penalties for each row(column) by taking the differences between the smallest and next smallest transportation cost entities in the same row (column) and write them in brackets against the corresponding row (column).

**Step 2.** Select the row or column with the largest penalty entities. If there is a tie in the values of penalties entities, then it can be broken by selecting the cell where the maximum allocation can be made.

**Step 3.** Allocate as much as possible in the lowest cost entities of the row( or column) which is defined by the **Step 2.**

**Step 4.** Adjust the supply and demand and cross-out the satisfied row or column.

**Step 5.** Repeat **Step 1.** **Step 2.** until the entire available supply at various sources and demand at various destinations are fully satisfied.
By MVAM, the IBFS of **Example 2.1.** is given as follows.

| | $D_1$ | $D_2$ | $D_3$ | *stock* |
|---|---|---|---|---|
| $O_1$ | $x_{11} = 5$ | $x_{12} = 45$ | | |
| | 7,10 | 5,7 | 8,12 | 50 |
| $O_2$ | $x_{21} = 0$ | | $x_{23} = 40$ | |
| | 6,8 | 9,12 | 7,9 | 40 |
| $O_3$ | $x_{31} = 30$ | | | |
| | 4,6 | 12,15 | 8,13 | 30 |
| *Demand* | 35 | 45 | 40 | |

**2.1.3. Determination of** $C_{ij}$

To solve this problem, apply our proposed algorithms stated as follows:

**2.1.4. Algorithm**

**Step 1.** Determine IBFS by by any of NWCM, MRMM, MCMM, MMMM, MVAM.

**Step 2.** After the allocate $x_{ij}$, determine $c_{ij}$ (unit transportation cost from source $O_i$ to destination $D_j$) as

$$c_{ij} = \begin{cases} \dfrac{\sum t_r R_{ij}(r)}{x_{ij}} & if \quad x_{ij} \neq 0 \\ 0 & if \quad x_{ij} = 0 \end{cases}$$

$$\begin{aligned} &where\ t_r \\ &are\ integer\ solution\ of \\ &\min \sum t_r R_{ij}(r) \\ &s.t.\ \ x_{ij} \le \sum t_r C_r \end{aligned}$$

**Step 3.** For non-basic cell $(i, j)$ possible allocation is the minimum of allocations in $i^{th}$ row and $j^{th}$
column (for possible loop). If possible allocation be $x_{ij}$, then for non-basic cell $c_{ij}$ (unit transportation
cost from source $O_i$ to destination $D_j$) as

$$c_{ij} = \begin{cases} \dfrac{\sum t_r R_{ij}(r)}{x_{ij}} & if \quad x_{ij} \neq 0 \\ 0 & if \quad x_{ij} = 0 \end{cases}$$

$$where\ t_r\ are\ integer\ solution\ of$$

$$\min \sum t_r R_{ij}(r)$$

$$s.t.\ \ x_{ij} \le \sum t_r C_r$$

In this manner we convert cost varying transportation problem to a usual transportation problem but $c_{ij}$ is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

**Step 4.** During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix $c_{ij}$ by **Step 2** and for non-basic we fix $c_{ij}$ by **Step 3**.

**Step 5.** Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

Using Algorithm(TP1), in **Example 2.1**, the unit transportation costs in basic cells (by MVAM) are $c_{11} = \frac{7}{5}$, $c_{12} = \frac{17}{45}$, $c_{21} = \frac{6}{5}$, $c_{23} = \frac{16}{40}$, $c_{31} = \frac{10}{30}$,

and the unit transportation costs in non-basic cells (by MVAM) are $C_{13} = \frac{8}{5}$, $C_{22} = \frac{21}{35}$, $C_{32} = \frac{10}{30}$ $C_{33} = \frac{21}{30}$

**2.1.5. Optimality test**

Once an initial basic feasible solution has been computed, the next step in the transportation problem is to determine whether the solution obtained in optimum or not. The method for testing optimality by cell evaluation is $u-v$ method.

$u-v$ **method:**

In order to test for optimality we should follow the $u-v$ method which is given bellow:

**Step 1.** Start with B.F.S. consisting of $m+n-1$ allocation in independent positions.

**Step 2.** Determine a set of $m+n$ numbers $u_i, i=1,\ldots,m$ and $v_j, j=1,\ldots,n$ such that in each cell $(i, j)$ $c_{ij} = u_i + v_j$

**Step 3.** Calculate cell evaluations (unit cost difference) $d_{ij}$ for each empty cell $(i, j)$ by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

**Step 4.** Examine the matrix of cell evaluation $d_{ij}$ for negative entries and conclude that

(i) If all $d_{ij} > 0,$ then Solution is optimal and unique.

(ii) If all $d_{ij} \geq 0$ and at least one $d_{ij} = 0$, then solution is optimal and alternative solution also exists.

(iii) If at least one $d_{ij} < 0$, then solution is not optimal.

If it is so, further improvement is required by repeating the above process after **Step** 5 and onwards.

**Step 5.** (i) See the most negative cell in the matrix $[d_{ij}]$.

(ii) Allocate $\theta$ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.

(iii) This value of $\theta,$ in general is obtained by equating to zero the minimum of the allocations containing $-\theta$ (not $+\theta$) only at the corners of the closed loop.

(iv) Substitute the value of $\theta$ and find a fresh allocation table.

**Step 6.** Again, apply the above test for optimality till we find $d_{ij} \geq 0$.

**Optimality test of Example 2.1 after MVAM and** $c_{ij}$

Determine a set of 6 numbers $u_i, i = 1,2,3$ and $v_j, j = 1,2,3$ such that in each cell basic $(i, j)$ $c_{ij} = u_i + v_j$, each non-basic cell $(i, j)$ by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1,2,3$, $v_j, j = 1,2,3$ and $d_{ij}$ non-basic cell $(i, j)$ is given in the following table

| | $D_1$ | $D_2$ | $D_3$ | $u_i$ |
|---|---|---|---|---|
| $O_1$ | $x_{11} = 5$ $c_{11} = \frac{7}{5}$ 7,10 | $x_{12} = 45$ $c_{12} = \frac{17}{45}$ 5,7 | $c_{13} = \frac{8}{5}$ 8,12 $d_{13} > 0$ | $\frac{7}{5}$ |
| $O_2$ | $x_{21} = 0$ $c_{21} = \frac{6}{5}$ 6,8 | $c_{22} = \frac{21}{35}$ 9,12 $d_{22} > 0$ | $x_{23} = 40$ $c_{23} = \frac{16}{40}$ 7,9 | $\frac{6}{5}$ |
| $O_3$ | $x_{31} = 30$ $c_{31} = \frac{10}{30}$ 4,6 | $c_{32} = \frac{10}{30}$ 12,15 $d_{32} > 0$ | $c_{33} = \frac{21}{30}$ 8,13 $d_{33} > 0$ | $\frac{10}{30}$ |
| $v_j$ | 0 | $-\frac{46}{45}$ | $-\frac{4}{5}$ | |

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{11} = 5, x_{12} = 45, x_{23} = 20, x_{32} = 40, x_{31} = 30$.

Minimum cost $Z^* = 7 + 17 + 16 + 10 = 50$ unit(Rs.)

The mathematical programming for CVTP under $N$ vehicle is described below.

### 2.1.6. Bi-level Mathematical Programming for CVTP under $N$ -Vehicle

The Bi-level mathematical programming for CVTP under $N$ -vehicle is formulated in **Model 1 (CVTPNV)** as follows:

**Model 1 (CVTPNV)**

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

*where, $c_{ij}$ is determinedby following mathematical programming*

$$c_{ij} = \begin{cases} \dfrac{\sum t_r R_{ij}(r)}{x_{ij}} & if \quad x_{ij} \neq 0 \\ 0 & if \quad x_{ij} = 0 \end{cases}$$

*where $t_r, r = 1, \ldots, N$*

*are integer solution of*

$$\min \sum t_r R_{ij}(r)$$

$$s.t. \;\; x_{ij} \le \sum t_r C_r$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1, \ldots, m;$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1, \ldots, n;$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$. \qquad (3)$$

$$x_{ij} \ge 0 \;\; \forall i, \; \forall j$$

**Theorem 2.1.** The number of basic variables in a balanced CVTP is at most $(m+n-1)$.

**Theorem 2.2.** A necessary and sufficient condition for the existence of a feasible solution to a CVTP is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

**Theorem 2.3.** The solution of a CVTP is never unbounded.

**Theorem 2.4.** A set $X$ of column vectors f coefficient matrix of a CVTP will be linearly dependent if their corresponding cells in the transportation tableau contain a loop.

### 2.2. Multi-Stage Transportation Problem

#### 2.2.1. Problem formulation

The transportation chain is of the form: supplier depots-entrepots-purchaser depots. The transportation in this problem can be optimized on two stages. It is supposed that $m$ supplier depots $(i = 1,\dots,m)$, $k$ entrepots $(k = 1,\dots,K)$ and $n$ purchaser depots $(j = 1,\dots,n)$. Then on the $1^{st}$ stage, transportation is done from the supplier depots to entrepots and on the $2$ nd stage the transportation is done from from the entrepots to the purchaser depots. And so on. The mathematical programming model for multi-stage transportation problem(MSTP) is given below.

**Model 2**

$$\min \sum_{l=1}^{L} \sum_{i=1}^{T_{(l-1)}} \sum_{j=1}^{T_l} c_{ij}^1 x_{ij}^1,$$

$$
\begin{aligned}
s.t. \quad & \sum_{j=1}^{T_{(l-1)}} x_{ij}^l = H_i^{(l-1)}, \; i = 1,\dots,T_{(l-1)} \\
& \sum_{i=1}^{T_l} x_{ij}^l = H_i^{(l)}, \; j = 1,\dots,T_l \\
& \sum_{i=1}^{T_{(l-1)}} H_i^{(l-1)} = \sum_{j=1}^{T_l} H_i^{(l)}, \forall l = 1,\dots,L \\
& x_{ij}^l \ge 0 \;\; \forall i, \; \forall j, \; \forall l
\end{aligned}
\tag{4}
$$

where

$m = T_0$: the number of suppliers $O_i, i = i,\dots,m$

$n = T_L$: the number of purchasers $D_j, j = 1,\dots,n$

$L$: the number of transportation stages $(l = 1,\dots,L)$

$L-1$: the number of entrepot stage

$a_i = H_i^0$: stock of $i^{th}$ suppliers $i = i,\dots,m$

$b_j = H_j^L$: demand of $j^{th}$ purchasers $j = 1,\dots,n$

$T_l$: the number of enterpots on $l^{th}$ stage $(E_1^l,\dots,E_{T_l}^l); l = 1,\dots,L-1$

$H_{T_l}^l$ capacity of the $T_l^{th}$ entrepot on $l^{th}$, $l = 1,\dots,L-1$ stage

$c_{ij}^l$ unit transportation cost on $l^{th}$ stage from $i^{th}$ supplier(entrepot) to $j^{th}$ entrepot(purchaser).

$x_{ij}^l$ transported amount on on $l^{th}$ stage from $i^{th}$ supplier(entrepot) to $j^{th}$ entrepot(purchaser).

### 2.3. Multi-Stage Transportation Problem under Vehicles

It is supposed that $m$ supplier depots $(i = 1,\dots,m)$, $k$ entrepots $(k = 1,\dots,K)$ and $n$ purchaser depots $(j = 1,\dots,n)$. Then on the $1^{st}$ stage, transportation is done from the supplier depots to entrepots and on the $2$nd stage the transportation is done from from the entrepots to the purchaser depots. And so on.

Again it is considered that the $c_{ij}^{1}$ is unknown and there are $N$-vehicle for transportation from the supplier depots to entrepots and on the $2$nd stage the transportation is done from from the entrepots to the purchaser depots. In each stage $l$, the fixed transportation cost for single trip of the vehicles are known. Then by methods of CVTP, the mathematical model of MSTP under vehicles is formulated as a Cost Varying Transportation Problem(CVMSTP). This model is represented in **Model 3**, which is a multi-stage bi-level mathematical model.

**Model 3**

$$\min \sum_{l=1}^{L}\sum_{i=1}^{T_{(l-1)}}\sum_{j=1}^{T_l} c_{ij}^{l} x_{ij}^{l},$$

$$\textit{where, } c_{ij}^{l} \textit{ is determined by following mathematical programming}$$

$$c_{ij}^{l} = \begin{cases} \dfrac{\sum t_r R_{ij}(r)}{x_{ij}^{1}} & \textit{if} \quad x_{ij}^{1} \neq 0, \\ & \qquad\qquad l = 1,\ldots,L \\ 0 & \textit{if} \quad x_{ij}^{l} = 0 \end{cases}$$

$$\textit{where } t_r,\ r = 1,\ldots,N \ \textit{ are integer solution of}$$

$$\min \sum t_r R_{ij}(r)$$

$$s.t.\ \ x_{ij}^{l} \le \sum t_r C_r \quad l = 1,\ldots,L$$

$$\sum_{j=1}^{T_{(l-1)}} x_{ij}^{l} = H_i^{(l-1)},\ i = 1,\ldots,T_{(l-1)}$$

$$\sum_{i=1}^{T_l} x_{ij}^{l} = H_i^{(l)},\ j = 1,\ldots,T_l$$

$$\sum_{i=1}^{T_{(l-1)}} H_i^{(l-1)} = \sum_{j=1}^{T_l} H_i^{(l)}, \forall l = 1,\ldots,L \tag{5}$$

$$x_{ij}^{l} \ge 0 \ \ \forall i,\ \forall j,\ \forall l$$

**3. Solution of CVMSTP**

Each stage of CVMSTP is a CVTP under $N$-vehicle. So we solve every CVTP under $N$-vehicle separately and determine their optimum values. Then sum up the optimal values and we get optimal solution of CVMSTP.4. Numerical Example.

**Example 2.** Suppose three suppliers $(O_1, O_2, O_3)$ with stocks $35,45,50$ boxes respectively, which are supplied to three supplier entrpots $(M_1^1, M_2^1, M_3^1)$ with demand $65,42,23$ boxes. Further on , boxes are transported to the four supplier entrepots $(M_1^2, M_2^2, M_3^2, M_4^2)$ with demand $40,35,25,30$ boxes. Lastly, boxes are transported to the four purchasers $(D_1, D_2, D_3, D_4)$ with demand $25,30,50,25$. Suppose there are two types of vehicles in each stage with carrying capacities $7$ boxes and $10$ boxes respectively, in a single trip. The transportation costs of vehicles in a single trip for each route and each stage are given in the following table.

| | $M_1^1$ | $M_2^1$ | $M_3^1$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|---|---|---|
| $O_1$ | 15,20 | 18,24 | 12,18 | | | | |
| $O_2$ | 10,15 | 13,17 | 20,25 | | | | |
| $O_3$ | 11,16 | 19,23 | 17,22 | | | | |
| | | | | | | | |
| $M_1^2$ | 5,10 | 9,13 | 10,15 | 10,13 | 5,9 | 12,16 | 15,21 |
| $M_2^2$ | 8,11 | 11,17 | 15,19 | 13,17 | 13,16 | 18,22 | 16,23 |
| $M_3^2$ | 13,17 | 12,15 | 20,23 | 19,23 | 14,19 | 9,13 | 8,14 |
| $M_4^2$ | 16,19 | 9,12 | 7,11 | 21,26 | 20,23 | 26,29 | 23,27 |

This MSTP under $2$-vehicle is split in the following three CVTP.

**Stage 1.** The CVTP from Supplier to $1^{st}$ entrepot is

| | $M_1^1$ | $M_2^1$ | $M_3^1$ | *Stock* |
|---|---|---|---|---|
| $O_1$ | 15,20 | 18,24 | 12,18 | 35 |
| $O_2$ | 10,15 | 13,17 | 20,25 | 45 |
| $O_3$ | 11,16 | 19,23 | 17,22 | 50 |
| *Demand* | 65 | 42 | 23 | |

Optimal solution of **stage** 1 *(see appendix 2)* is $x_{11}^1 = 12$, $x_{13}^1 = 23$, $x_{21}^1 = 3$, , $x_{22}^1 = 42$, $x_{31}^1 = 50$,

Minimum cost is this stage is $Z^{1*} = 30 + 42 + 10 + 77 + 80 = 239$ unit(EURO).

**Stage 2.**

The CVTP from $1^{st}$ entrepot to $2nd$ entrepot is

Optimal solution of **stage** 2 *(see appendix 2)* is $x_{11}^2 = 40$, $x_{12}^2 = 25$, $x_{22}^2 = 10$, $x_{23}^2 = 25$, $x_{24}^2 = 7$, $x_{34}^2 = 23$,

Minimum cost is this stage is $Z^{2*} = 30 + 30 + 17 + 42 + 9 + 25 = 153$ unit(EURO).

**Stage 3**

The CVTP from $2nd$ entrepot to purchasers is

| | $M_1^2$ | $M_2^2$ | $M_3^2$ | $M_4^2$ | *Stock* |
|---|---|---|---|---|---|
| $M_1^2$ | 10,13 | 5,9 | 12,16 | 15,21 | 40 |
| $M_2^2$ | 13,17 | 13,16 | 18,22 | 16,23 | 35 |
| $M_3^2$ | 19,23 | 14,19 | 9,13 | 8,14 | 25 |
| $M_4^2$ | 21,26 | 20,23 | 26,29 | 23,27 | 30 |
| *Demand* | 25 | 30 | 50 | 25 | |

Optimal solution of **stage** 3 *(see appendix 2)* is $x_{12}^3 = 30$, $x_{13}^3 = 10$, $x_{21}^3 = 20$, $x_{23}^3 = 15$, $x_{33}^3 = 25$, $x_{41}^3 = 5$, , $x_{44}^3 = 25$,

| | $M_1^2$ | $M_2^2$ | $M_3^2$ | $M_4^2$ | *Stock* |
|---|---|---|---|---|---|
| $M_1^1$ | 5,10 | 8,11 | 13,17 | 16,19 | 65 |
| $M_2^1$ | 9,13 | 11,17 | 12,15 | 9,12 | 42 |
| $M_3^1$ | 10,15 | 15,19 | 20,23 | 7,11 | 23 |
| *Demand* | 40 | 35 | 25 | 30 | |

Minimum cost is this stage is $Z^{3*} = 24 + 16 + 34 + 30 + 35 + 21 + 77 = 237$ unit(EURO).

Therefore the CVMSTP ( **Example 2**) is

$x_{11}^1 = 12$, $x_{13}^1 = 23$, $x_{21}^1 = 3$, , $x_{22}^1 = 42$, $x_{31}^1 = 50$, $x_{11}^2 = 40$, $x_{12}^2 = 25$, $x_{22}^2 = 10$, $x_{23}^2 = 25$, $x_{24}^2 = 7$, $x_{34}^2 = 23$, $x_{12}^3 = 30$, $x_{13}^3 = 10$, $x_{21}^3 = 20$, , $x_{23}^3 = 15$, $x_{33}^3 = 25$, $x_{41}^3 = 5$, , $x_{44}^3 = 25$,

$$\text{Min Cost } Z^* = Z^{1*} + Z^{2*} + Z^{3*} = 239 + 153 + 237 = 629 \text{ units (EURO).}$$

**4. Conclusions**

The proposed method provides a solution for MSTP under finite number of vehicles. Any TP under finite number of vehicles is transform to a CVTP. The IBFS of CVTP are determined by our modified methods namely, MRMM, MCMM, MMMM and MVAM. Since in CVTP the unit transportation cost is unknown, we determine varying unit transportation cost by our proposed algorithm TP1. To determine the varying unit transportation cost, it is needed to know all information(cost of single trip) of all vehicles in all allocations. Optimality test is nearly similar in a TP. We develop the bi-level mathematical programming not only CVTP but also for CVMSTP after constructing the model of MSTP. Since each stage of CVMSTP is a CVTP, so we solve the CVMSTP by help of CVTP. But further study is needed when the balanced condition (1) is not satisfied.

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