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Development of a Simple Theorem in Solving Transportation Problems

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ABSTRACT

This paper deals with developing an effective theorem related to solving transportation problems. In view of this theorem one can find out the nature of the optimal solution and how much the total transportation cost is changed after adding (or subtracting) a constant quantity to each unit cost in the transportation cost matrix. A numerical example is presented to illustrate and verify the theorem.

Keywords: Transportation problem, initial basic feasible solution, optimal solution, transportation cost matrix.

1. Introduction

The transportation problem (TP) is a special type of linear programming problem which deals with shipping a single homogeneous commodity from several sources (e.g. plants, factories etc.) of supply to various destinations (e.g. warehouses, markets etc.) of demand in such a way that the total transportation cost is a minimum. It has wide applications in Management Sciences, Engineering and Technology. The model can be extended in a direct manner to cover practical situation in the areas of inventory control, employment scheduling, personnel assignment, cash flow statements and many others. The basic transportation problem was first introduced by Hitchcock [1] and further developed by Koopmans[2]. As it is basically a linear programming problem it can be solved by regular simplex method. In 1951 Dantzig [3] solved it by simplex method. However this method is complex and inefficient especially for large scale transportation problem. Thus the special structure of the TP allows the development called transportation technique that is computationally more efficient. There are two stages in transportation technique: finding initial basic feasible solution (IBFS) and testing the solution for optimality. In literature several heuristic methods are available to obtain initial basic feasible solution, such as North-West Corner Rule, Least Cost Method, Row Minima, Column Minima, Vogel Approximation Method (VAM) [4], Russell Approximation Method [5] etc. For testing optimality of the initial basic feasible solution, Modified Distribution Method (MODI) is frequently used. Charnes and Cooper [6] developed the Stepping Stone Method which provides an alternative way of determining the optimal solution. In 1990 Kirca and Satir [7] developed a heuristic, called TOM (Total Opportunity Cost Method) and they used Least Cost Method with some tie breaking rules on the TOC matrix for generating an IBFS to the TP. Mathirajan and Meenakshi [8] extended the idea of Kirca and Satir using VAM procedure. Nagoor Gani and Abdul Razak [15] proposed a parametric approach for two stage fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Recently a literature search revealed a few heuristic methods such as Improved VAM (IVAM) [9], Zero suffix method [10], ASM [11], Zero point method [12] and Average Cost Method (ACM) [13] for finding optimal solution. Pandian and

Natarajan [14] proposed a new method for finding an optimal more-for-less (MFL) solution for transportation problems with mixed constraints. In this paper we develop a theorem which provides the nature of the optimal solution and the variation of the total transportation cost under certain imposed conditions.

2. Mathematical formulation

Let a_i be the number of supply units required at source $i(i = 1,2,........,m)$, b_j be the number of demand units required at destination $j(j = 1,2,........,n)$ and c_{ij} represents the unit transportation cost for transporting the units from source i to destination j. If x_{ij} is the number of units shipped from source i to destination j, the equivalent linear programming model will be

$$\text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for } i = 1,2,........,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for } j = 1,2,........,n$$

$$x_{ij} \geq 0 \quad \forall\, i, j$$

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations.

$$\text{i.e.} \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \,.$$

Otherwise, it is called unbalanced. The balanced condition is both necessary and sufficient condition for the existence of a feasible solution to the transportation problems. In matrix form the transportation problem can be summarized as

	D_1	D_2	…	D_n	Supply
O_1	x_{11} c_{11}	x_{12} c_{12}	…	x_{1n} c_{1n}	a_1
O_2	x_{21} c_{21}	x_{22} c_{22}	…	x_{2n} c_{2n}	a_2
⋮	⋮	⋮	…	⋮	⋮
O_m	x_{m1} c_{m1}	x_{m2} c_{m2}	…	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	…	b_n	

3. Proposed theorem

Theorem 1. If a constant k is added (or subtracted) to each entry in the p-th row in a transportation matrix $[c_{ij}]$ then the optimal solution remains unaltered and the total transportation cost is increased (or decreased) by $k \times$***supply at p-th source*****.**

Proof:

Let $\{x_{ij} : i = 1, 2, ..., m;\ j = 1, 2, ..., n\ \}$ be the optimal solution with respect to the original cost matrix $\lfloor c_{ij} \rfloor$. Then the total cost (objective function),

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

After adding (or subtracting) a constant k to each element in the p-th row of the cost matrix $\lfloor c_{ij} \rfloor$, the new total cost (objective function) becomes

$$\begin{aligned} z' &= \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \,(i \neq p) + \sum_{j=1}^{n} (c_{pj} \pm k) x_{pj} \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \,(i \neq p) + \sum_{j=1}^{n} c_{pj} x_{pj} \pm k \sum_{j=1}^{n} x_{pj} \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \pm k.a_p \\ &= z \pm k.a_p \end{aligned}$$

where a_p =supply at p-th source.

Here $k.a_p$ is constant (independent of x_{ij}). Thus if z is minimum for a set of values of $\{x_{ij}\}$ of the original problem then z' is also minimum for the same set of values of $\{x_{ij}\}$ of the new problem as z' differs from z only by a constant. Hence the total transportation cost is increased (or decreased) by $k.a_p$.

In a like manner, it can be shown that on adding (or subtracting) a constant k to every cost element of the q-th column in a transportation matrix $\lfloor c_{ij} \rfloor$, the optimal solution remains the same, while the total transportation cost is increased (or decreased) by $k.b_q$, where b_q =demand at q-th destination.

By virtue of Theorem 1, we can develop the following Theorem in a general form.

Theorem 2. If a constant is added (or subtracted) to each element of the transportation cost matrix, an optimal solution of the original problem remains optimal for the new problem; and the total transportation cost is increased (or decreased) by ***constant×total supply(or total demand).***

Proof: Let $\{x_{ij} : i = 1, 2, ..., m;\ j = 1, 2, ..., n\ \}$ be the optimal solution with respect to the original cost matrix $\lfloor c_{ij} \rfloor$. Then the total cost (objective function),

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

If a constant k is added (or subtracted) to each element of the cost matrix $\lfloor c_{ij} \rfloor$ and z' denotes the total cost for the modified cost matrix $\lfloor c_{ij} \pm k \rfloor$ then

$$z' = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} \pm k) x_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \pm k \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = z \pm k. \sum_{i=1}^{m} a_i$$

$$= z \pm k.\sum_{i=1}^{m} a_i = z \pm k.\text{total supply (or total demand)}$$

Since k. *total supply* is constant (independent of x_{ij}), it is clear that every optimal solution $\{x_{ij}\}$ corresponding to the matrix $\lfloor c_{ij} \rfloor$ is also an optimal solution corresponding to the matrix $\lfloor c_{ij} \pm k \rfloor$ and vice versa, i.e. upon adding (or subtracting) a *constant* to every cost element of a transportation problem, the optimal solution remains unaltered, while the total transportation cost is increased (or decreased) by ***constant*×*total supply(or total demand).***

4. Numerical example

To illustrate the theorem we consider the following transportation problem:

	D_1	D_2	D_3	D_4	Supply
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	

Solving the problem using Vogel's Approximation Method (VAM), the optimal transportation table is presented below:

	D_1	D_2	D_3	D_4	Supply
O_1	[5] 19	30	50	[2] 10	7
O_2	70	[2] 30	[7] 40	60	9
O_3	40	[6] 8	70	[12] 20	18
Demand	5	8	7	14	

Hence the optimal solution and the total transportation cost are

$$x_{11} = 5,\ x_{14} = 2,\ x_{22} = 2,\ x_{23} = 7,\ x_{32} = 6,\ x_{34} = 12 \quad and \quad z = 743.$$

If we add 10 to each unit cost in the original transportation problem, the modified matrix becomes

	D_1	D_2	D_3	D_4	Supply
O_1	29	40	60	20	7
O_2	80	40	50	70	9
O_3	50	18	80	30	18
Demand	5	8	7	14	

Solving the problem using Vogel's Approximation Method (VAM), the optimal transportation table is presented below:

	D_1	D_2	D_3	D_4	Supply
O_1	[5] 29	40	60	[2] 20	7
O_2	80	[2] 40	[7] 50	70	9
O_3	50	[6] 18	80	[12] 30	18
Demand	5	8	7	14	

Hence the optimal solution and the total transportation cost are

$$x_{11} = 5,\ x_{14} = 2,\ x_{22} = 2,\ x_{23} = 7,\ x_{32} = 6,\ x_{34} = 12 \quad and \quad z' = 1083.$$

Thus we see that after adding a constant (10) to each cost element in the original cost matrix, the optimal solution remains unchanged and the total transportation cost is increased by 10×34(=constant $\times$ total supply)=340.

Again if we subtract a constant (6) from each unit cost in the original transportation problem, the reduced matrix is as follows:

	D_1	D_2	D_3	D_4	Supply
O_1	13	24	44	4	7
O_2	64	24	34	54	9
O_3	34	2	64	14	18
Demand	5	8	7	14	

Solving the problem using Vogel's Approximation Method (VAM), the optimal transportation table is presented below:

	D_1	D_2	D_3	D_4	Supply
O_1	[5] 13	24	44	[2] 4	7
O_2	64	[2] 24	[7] 34	54	9
O_3	34	[6] 2	64	[12] 14	18
Demand	5	8	7	14	

Hence the optimal solution and the total transportation cost are

$$x_{11} = 5,\ x_{14} = 2,\ x_{22} = 2,\ x_{23} = 7,\ x_{32} = 6,\ x_{34} = 12 \quad and \quad z' = 539.$$

Thus it is observed that on subtracting a constant (6) from each cost element in the original cost matrix, the optimal solution remains the same and the total transportation cost is decreased by 6×34(=constant $\times$ total supply)=204.

Hence the theorem is verified.

5. Conclusion

In this paper an attempt has been made to develop a new theorem in case of solving transportation problems and proved it in a simple and easy way. We have also verified the theorem with a numerical example. It can be an important tool for the decision makers who are handling distribution and logistics related problems by aiding them in the decision making process and providing an optimal solution in a simple and effective manner.

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