2014

M.Sc. Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VIII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

[Mathematical Methods]

[Marks : 50]

Answer *Q. No. 1* and any *three* from the rest.

1. Answer any *two* of the following : 4×2

(a) Define Starm — Liouvill problem involving boundary value problem. Write its important properties.

(b) For each of the following functions, determine which has a Laplace. If it exists, find it, if it does not, say briefly why?

(i) $e^{1/t}$

(ii) $f(s) = \begin{cases} 1, \text{ if t is even} \\ 0, \text{ if t is odd} \end{cases}$

(c) Find Fourier transform of $e^{-\frac{x^2}{2}}$ and hence find the function whose fourier transform is $e^{-\frac{\alpha^2}{2}}$

2. (a) Find the Laplace transform of $J_o(t)$, where $J_o(t)$ is Bessel's function of order zero.

(b) Prove that $H_n\left\{\frac{d^2 f}{dr^2}+\frac{1}{r}\frac{dt}{dr}-\frac{n^2}{r^2}f\right\}=-\alpha^2.f_n(\alpha)$;

provided both $rf^1(r)$ and $rf(r)$ tend to zero as $r \to 0$ and $r \to \infty$ where Hn stands for nth order Hankel transforms.

(c) Prove that the Fourier transform of $\frac{1}{n}$ is $i\sqrt{\frac{\pi}{2}}\,\text{sgn}(\alpha)$, where sgn is a signum function.

5+6+3

3. (a) State and prove computation theorem concerning on Laplace transform.

(b) Give an example to show that the integral of a good function is not necessarily a good function.

(c) Find the solution of the following problem of free vibration of a stretched string of an infinite length.

PDE : $\frac{\partial^2 u}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=0, \quad -\infty < x < \infty$,

BCs : $u(x, 0) = f(x)$

$\frac{\partial}{\partial t}u(x, 0) = h(x)$

u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$.

5+2+7

4. (a) Use Green's function technique to solve

$\frac{-\partial^2 u}{\partial x^2} - 4u = f(x); \quad 0 \le x \le 1$ with the boundary conditions $u(0) = u(1) = 0$.

(b) Discuss the solution procedure of homogeneous Fredholm integral equation of the second kind with degenerate Kernel.

(c) Find the first order Hankel transform of $f(r) = \frac{1}{r}e^{-ar}$

6+5+3

5. (a) If λ is an eigen value of the regular starm – Lionvilla problem.

$$[p(x)y'(n)]' + y(x)[q(n)\lambda r(n)] \cdot = 0, \ a < x < b$$

Subject to $\alpha_1 y(a) + \alpha_2 y^1(a) = 0$

$$\beta_1 y(a) + \beta_2 y^1(a) = 0$$

then show that λ must be a real number.

(b) Reduce the Voltera Integral Equation of first kind.

$x = \int_0^x Cos(x-t)\ y\ (t)\ dt$ to an Voltera integral equation of the second kind and the solve it.

(c) If the function f(t) has period T>0, show that Laplace transform of f(t) is $\frac{1}{1-e^{-pT}} \int_0^T e^{-pT}$ f(t) dt.

6+5+3

6. (a) Reduce the BvP $\frac{d^2y}{dx^2} + \lambda ny = 1$, in $0 \le x \le 1$ with boundary conditions y(o) = 0, y(ℓ) = 1 to an integral equation and find Kernel. 6

(b) If L{f(t)} is the Laplace transform of a function f(t), which is piecewise continuous in any finite interval of t and is of exponential order $O(e^{\lambda t})$ at $t \to \infty$, show that

$$\underset{b \to \infty}{Lt}\ bf(p) = f(o).$$ 4

(c) Define finite Hankel transform of order n of a function of order n of a function f(r), $o \le r \le a$ and state its inversion formula. Find the zero-order Hankel transform of e^{-ar}, a > 0 of in simplest form. 4

Group—B

[Dynamical Oceanology and Meteorology for students whose special paper is OR]

[Marks : 50]

Answer Q. No. 12 and any three for the rest.

7. (a) Find the rate of change circulation in the atmosphere and interpret each term. 8

(b) Define the potential temperature and show that it is invariant during the adiabatic motion in the atmosphere. 4

(c) Deduce the geostrophic wind equation in the atmosphere. 4

8. (a) Derive the equivalence of Emagram and discuss its different properties. 8

(b) Derive the expression of the pressure gradient force in the atmosphere. 4

(c) Discuss different types of fronts in the atmosphere. 4

9. (a) Establish Gibbs–relation of thermodynamics. Deduce Gibbs–Duhem relation for sea water.

(b) Find the condition of stability of equilibrium of a stratified fluid and hence explain the significance of *Brünt – Väisälä* frequency. 8+8

10. Explain β-plane approximation. Assuming the sea-water to be a non-viscous strafied fluid, deduce the β-plane equation and examine the range of validity of these equations. 8+8

11. (a) Express the principle of conservation of mass in the form of following pair of equations.

$$\frac{D\rho}{Dt} + \rho \, div \, \vec{q} = 0, \quad \rho \frac{Ds}{Dt} + - \, div \, \vec{I}_s, \text{ in usual notation,}$$

where sea-water is assumed to be a two component mixture of salt and pure water.

(b) Deduce the equation of state for moist air in the following from $p\alpha = R_4 T\left[1-(I-e)\frac{e}{p}\right]$. Hence define the virtual temperature and show that the virtual temperature is always greater than the actual temperature. 7

12. Define virtual temperature. 2

Group—B

(Elements of optimization and operations Research)

[For the students whose special paper is OM]

[Marks : 50]

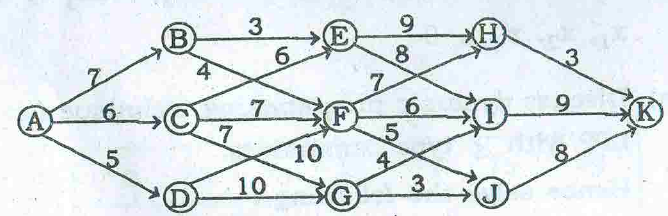
Answer Q. No. 7 and any three from the rest.

7. State Bellman's principle of optimization ? 2

Or

Define lead time and demand in connection with inventory model. 2

8. (a) Find the shortest path from vertex A to vertex K along the arcs joining the various vertices lying between A and K, as shown in the following figure.



8

(b) Discuss the effect of discrete change in the requirement vector b to the following L.P.P.

Maximize z = Cx

Subject to Ax = b, $x \geq 0$

Where $c, x^T \in R^n$, $b^T \in R^m$ and A is an $m \times n$ real matrix.

9. (a) A particular item has demand of 9000 units/year. The cost of procurement is Rs. 100 and the holding cost is Rs. 2.40 per unit per year. The replacement is instantaneous and no shortages are allowed. Determine

(a) The economic lot size;

(b) The time between orders;

(c) The number of order per year;

(d) The total cost per year if the cost of one unit is Rs. 1.

2+2+2+2

(b) Solve the following problems by using the Gomory's method

Max $z = x_1 + 2x_2 + x_3$

Subject to constraint $2x_1 + 3x_2 + 3x_3 \leq 11$ and

$x_1, x_2, x_3 \geq 0$ 8

10. (a) Discuss dynamic programming technique for solving LPP with $\leq$ type constraints.

Hence solve the following :

Maximize $z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 1$, $3x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$.

8

(b) What is quadratic programming problem? Discuss the Wolfe's Method to solve the quadratic programming problem. 2+6

11. (a) Solve the following LPP by using revised simplex method

Maximize $z = x_1 + 2x_2$

Subject to $2x_1 + 5x_2 \geq 6$

$x_1 + x_2 \geq 2$

and $x_1, x_2 \geq 0$

8

(b) Find the optimum order quantities for a multi-item EOQ model with investment constraint.

12. (a) The optimal solution of the LPP

Maximize $z = 3x_1 + 5x_2$

Subject to $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 1$

and $x_1, x_2 \geq 0$

is contained in the table

c_B	y_B	x_B	y_1	y_2	y_3	y_4
0	y_3	$\frac{2}{3}$	$\frac{1}{3}$	0	1	$-\frac{1}{3}$
5	y_2	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$
		$\frac{5}{3}$	$\frac{1}{3}$	0	0	$\frac{5}{3}$

Find the ranges of C_1 and C_2 for which the optimal solution remain optimal solution when changes one at a time.

4+4

(b) Solve the following quadratic programing problem by Beale's method.

$$\text{Maximize } z = 2x_1 + 3x_2 - x_1^2$$

$$\text{Subject to} \quad x_1 + 2x_2 \leq 4$$

$$\text{and} \quad x_1, x_2 \geq 0.$$