2014

M.Sc. Part-I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IV

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A
(*Principles of Mechanics*)

[Marks : 50]

Answer *Q. No. 1* and any *three* questions from the rest.

1. Answer any *one* question : 2

(a) What do you mean by holonomic and non-holonomic constraints ? Give examples each of these constraints.

(Turn Over)

(b) What do you mean by generalized force ? Find the expression of it in terms of generalised coordinates.

2. (a) What is the effect of the Coriolis force on a particle falling freely under the action of gravity ?

(b) Show that with respect to a uniformly rotating reference frame Newton's second law for a particle of mass m acted upon by real force $\vec{F}$ can be expressed as :

$$\vec{F}_{eff} = \vec{F} - 2m\vec{w} \times \vec{V}_{rot} - m\vec{w} \times (\vec{w} \times \vec{r})$$

Assume that the origins of the inertial and non-inertial coordinates systems are coincident. $\vec{F}_{eff}$ and $\vec{V}_{rot}$ represent respectively the effective force and velocity in rotating frame. 8+8

3. (a) If all the co-ordinates of a dynamical system of n degrees of freedom are ignorable, prove that the problem can be solved completely by integration.

(b) The Lagrangian for a coupled harmonic oscillator is given by :

$$L = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2) - \frac{1}{2}(\omega_1^2 q_1^2 + \omega_2^2 q_2^2) + \alpha q_1 q_2'$$

where α, ω_1, ω_2 are constants and q_1, q_2 are suitable co-ordinates. Find the Hamiltonian of the system. Write down the Lagrange's equations. 8+8

4. (a) Prove that

$$J = \int_{x_6}^{x_1} F(y_1, y_2, \ldots, y_n, y_1', y_2', \ldots, y_n', x)\, dx$$

will be stationary only if

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y_j'}\right) - \frac{\partial F}{\partial y_j} = 0, \; j = 1, 2, \ldots, n$$

where $y_j' = \frac{\partial y_j}{\partial x}$. 8

(b) What do you mean by Canonical Transformation ? If the generating function is given then show that the canonical transformation can be determined from it. What is the relation between the old and new Hamiltonians, when the generating function is explicitly independent of time ? 2+5+1

5. (a) Define Poisson bracket. If [X, Y] denotes the Poisson bracket of two dynamical variables X and Y, then show that :

$$\frac{\partial}{\partial x}[X, Y] = \left[\frac{\partial X}{\partial x}, Y\right] + \left[X, \frac{\partial Y}{\partial x}\right].$$ 4

(b) Find the condition that the transformation $P = ap + bq$, $Q = cp + dq$ is canonical, where a,b,c,d are constants. 4

(c) Solve the Harmonic oscillation problem by Hamilton-Jacobi method. 8

6. (a) Discuss a method to determine the eigen frequencies and normal modes of small oscillation of a dynamical system. 8

(b) Let m_0 be the mass of a particle at rest and m be the mass of the same particle when it is moving with velocity v. Then show that in relativistic mechanics

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where c is the velocity of light.

8

Group—B

(Partial Differential Equation)

[Marks : 50]

Answer *Q. No. 1* and any *three* from the rest.

1. What are the differences between ordinary differential equation and partial differential equation. 2

Or

Define order and degree of a partial differential equation. 2

2. (a) Find the integral surface of the linear PDE

$$x^2p + y^2q + z^2 = 0$$

which passes through the hyperboloid

$xy = x + y,\ z = 1.$ 8

(b) Solve : $p^3 - q = 0,\ u(x, 0) = 2x\sqrt{x},\ 0 \le x \le 1.$

State Cauchy-Kowalewski theorem. 5+3

3. (a) Determine the general solution of

$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$ 8

(b) Solve :

$$\left(D^2 - D'^2 + D - D'\right)z = e^{2x+3y}.$$ 8

4. (a) For an infinitely long conducting cylinder of radius 'a', with its axis coincidence with Z-axis, the voltage $u(r, \theta)$ obeys the Laplace's equation :

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\quad 0 \le r \le \infty,\ 0 \le \theta \le 2\pi$$

Find the voltage $u(r, \theta)$ for $r \ge a$ if $\lim_{r\to\infty} u(r,\theta) = 0$

subject to the condition $\frac{\partial u}{\partial r} = \frac{1}{a}u_0 \sin 3\theta$ at $r = a$.

8

(b) State and prove Mean-value theorem for harmonic function.

Also, show that if a harmonic function vanishes at all points on the boundary, then it is identically zero everywhere. 5+3

5. (a) Using the method of separation of variables, solve the following : 8

$u_{tt} - 4u_{xx} = 0,\ 0 < x < 1,\ t > 0$

$u_x(0, t) = u_x(1, t) = 0,\ t \geq 0$

$u(x, 0) = \cos^2(\pi x),\ 0 \leq x \leq 1$

$u_t(x, 0) = \sin^2(\pi x)\cos(\pi x),\ 0 \leq x \leq 1$

(b) Define the following : 4×2

(i) Domain of dependence ;

(ii) Domain of influence ;

(iii) Neumann problem for Laplace equation ;

(iv) First order semi-linear partial differential equation.

6. (a) Establish the Poisson integral formula of the interior Dirichlet Problem for a circle. 8

(b) If $\Delta u = 0$ in D and $\frac{\partial u}{\partial n} = 0$ on γD, then show that u is constant in D. 4

(c) Find the value of $\frac{1}{(xD+1)}\left(\frac{1}{x}\right)$, where $D = \frac{d}{dx}$. 4